The innovator's risk premium: Sticky hurdle rates, the cost of capital, and creative destruction*

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Abstract

Firms' hurdle rates exceed their financial cost of capital. This gap varies across firms and has widened in the aggregate. In our Schumpeterian model, two frictions drive this gap: firm decision-makers require an innovator's risk premium for undiversifiable innovation risk, and innovation profits are imperfectly pledgeable. Higher undiversifiable risk lowers the financial cost of capital while hurdle rates remain sticky or even rise. Widening gaps weaken creative destruction; profits and superstar valuations rise despite firms forgoing positive-NPV projects. The calibrated model matches cross-sectional patterns in gaps, R&D, market power, and firm dynamics, and explains weak productivity growth and declining dynamism.

Keywords: hurdle rates, cost of capital, idiosyncratic risk, innovation, market power, firm valuations

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Firms' hurdle rates (the discount rates they use to evaluate investment) exceed their cost of capital in financial markets. This gap varies across firms and has widened in the aggregate (Gormsen and Huber 2025): the financial cost of capital has fallen in recent decades but reported hurdle rates have not declined commensurately and realized returns on business capital have increased moderately.[1] Over the same period, productivity growth, entry, and business dynamism have slowed, while markups, profits, and firm valuations have risen.[2] These patterns raise questions at the intersection of finance and growth. When firms and financial markets discount innovation differently, how do the resulting wedges shape creative destruction and thereby affect productivity growth, market competition, and firm valuations? And how can a falling financial cost of capital coexist with sticky hurdle rates, weak growth, and slowing dynamism, amid rising profits and firm values?

We develop a Schumpeterian model in which the hurdle rates governing firms' R&D decisions differ from the cost of capital at which financial markets value their cash flows. The wedge arises endogenously from two frictions in innovation investment: firm decision-makers bear undiversifiable idiosyncratic risk, and innovation profits are imperfectly pledgeable. We embed these frictions in a creative-destruction environment with endogenous market power and strategic competition between market leaders and laggards (Aghion et al. 2001; Akcigit and Ates 2023).

These frictions generate a falling cost of capital and sticky hurdle rates when undiversifiable idiosyncratic risk rises. Higher risk depresses the financial cost of capital

[1] Internet Appendix A describes the evolution of the return on capital, using the approach of Caballero et al. (2017) and Farhi and Gourio (2018).

[2] Akcigit and Ates (2021) summarize and discuss these trends. Regarding valuations, see Eggertsson et al. (2021), Kuvshinov and Zimmermann (2022), and Cho et al. (2026).

through greater precautionary saving and weaker productivity growth. At the same time, it increases the innovator's risk premium: the compensation that firm decision-makers require for bearing innovation risk. This raises the required return on innovation directly and, because innovation profits are imperfectly pledgeable, tightens financing constraints that further raise hurdle rates. Hurdle rates therefore do not fall in step with the financial cost of capital: in our calibrated model, hurdle rates rise moderately even as the financial cost of capital falls notably.

This asymmetric adjustment widens the discount-rate gap and weakens creative destruction. Higher hurdle rates reduce innovation by leaders and laggards; when we extend the model to include entrants, entry also falls. Because laggards' and entrants' innovations tend to be more-than-incremental, weaker laggard and entrant innovation makes leadership more durable, widens technology gaps, and reduces competition.[3] The calibrated model therefore generates slower productivity growth, higher markups, higher profitability, and weaker business dynamism, alongside higher firm valuations, especially for superstars. Valuations rise even as growth falls and firms forgo positive-NPV R&D: weakening creative destruction raises profits, and the lower financial cost of capital raises their value. Thus, the model reconciles a falling financial cost of capital and rising firm values with sticky hurdle rates and weak innovation.

The calibration matches aggregate and cross-sectional moments related to hurdle rates, the financial cost of capital, innovation output, productivity growth, markups,

[3] Laggards' and entrants' mix of incremental and more significant innovations is parameterized in the model; it is also an outcome of the model calibration, disciplined by moments related to business dynamism and creative destruction. As described later in the introduction, the calibrated model reproduces relevant, nontargeted firm-dynamics patterns such as the higher and more volatile growth of smaller firms. For evidence that small innovative firms and new entrants have a comparative advantage in achieving significant advances, see also Akcigit and Kerr (2018) and the discussion in Acemoglu and Cao (2015).

profit volatility, and R&D intensity. It also captures key features of the full, nontargeted distributions of markups, innovation output, and discount-rate gaps. Our model shows how greater exposure to undiversifiable risk can arise from increased sensitivity of decision-makers' payoffs to idiosyncratic performance, even if raw idiosyncratic risk is unchanged. Such increased exposure is consistent with the emergence of high-powered, equity-based managerial incentives among public firms and with the migration of innovative activity toward private firms in which insiders have large ownership stakes.[4]

Our framework generates distinct cross-sectional predictions. First, in both the data and model-simulated panels, higher discount-rate gaps are associated with lower future R&D, after absorbing firm and time effects. Second, the determinants of gaps line up with the data: firms with greater initial market power experience larger increases in discount-rate gaps, more financially constrained firms have higher gaps, and firms with more volatile idiosyncratic returns have higher gaps. Third, the model reproduces the persistence of market leadership and discount-rate gaps. These patterns support interpreting discount-rate gaps as shaped by financing constraints, risk, and firms' positions in the creative-destruction process.

Creative destruction is central to our analysis of growth and market competition. A higher hurdle rate reduces the present value of innovation profits, but in a strategic environment it also changes which firms innovate and how the distribution of leader-laggard technology gaps evolves. These composition effects shape aggregate growth and markups (Acemoglu and Akcigit 2012; Cavenaile et al. 2025) and can amplify, dampen, or—as shown in Liu, Mian and Sufi (2022)—even overturn the standard channel through

[4]These trends are discussed in Section 4 and Internet Appendix B.

which a higher discount rate reduces growth by lowering the discounted value of innovation profits. Because transmission depends on the creative-destruction process, we assess our model's performance along these margins. The model matches the contribution of reallocation to productivity growth in the Foster et al. (2001) decomposition; it reproduces the higher and more volatile growth of smaller firms; and, consistent with Kogan et al. (2017), a firm's own innovation raises its future profits while competitor innovation lowers them. These nontargeted moments increase confidence that the framework captures the leader-laggard, firm-size, and reallocation margins through which discount-rate gaps affect growth, market power, and business dynamism.

We also distinguish our mechanism from related explanations for weak growth and declining dynamism. Shocks to product-market power create static production wedges rather than an intertemporal wedge between hurdle rates and the financial cost of capital. Absent our two frictions, such shocks leave the discount-rate gap at zero. Moreover, in our calibrated model, higher pricing power raises productivity growth, in contrast to the observed decline, and, if anything, slightly raises the cost of capital. Entry barriers reduce entry but do not by themselves generate discount-rate gaps for continuing firms; in the calibrated model, they lower the aggregate discount-rate gap, with little effect on productivity growth or the financial cost of capital.[5] A higher equity premium introduces a different wedge, between the financial cost of capital and the risk-free rate. By raising the financial cost of capital, it lowers valuations, contrary to the data. Thus, while these forces may matter for particular outcomes, they do not reproduce the cross-sectional

[5]Higher pricing power can raise productivity growth by increasing the incentive to innovate and attain market leadership. Entry barriers can have little effect on productivity growth if the lower risk of displacement induces market leaders to innovate more, offsetting a lower contribution to growth from entrants.

evidence on discount-rate gaps and the aggregate trends in hurdle rates, the financial cost of capital, growth, entry, markups, and valuations.

Finally, we use the framework to connect with the ideas-hard-to-find literature. The firm-level evidence above supports the model's channels linking hurdle rates and R&D. At the aggregate level, however, the R&D to GDP ratio has risen while productivity growth has fallen, a pattern often described as ideas becoming harder to find (Bloom et al. 2020). In our framework, a higher discount-rate gap by itself tends to reduce both research effort and growth, so the aggregate divergence points to concurrent changes in how ideas are produced. We therefore allow for changes in idiosyncratic risk as well as in laggard catch-up and the benefits of scale in R&D.[6] We choose the direction and magnitude of these changes to match observed movements in the discount-rate gap, productivity growth, markups, and R&D to GDP. The calibration points to greater exposure to undiversifiable idiosyncratic risk, weaker laggard catch-up, and a more scale-intensive R&D technology. Together, these changes generate slower productivity growth despite rising aggregate R&D, while also producing a moderate increase in hurdle rates, a sharper decline in the cost of capital, and weaker business dynamism. Higher idiosyncratic risk is needed for the discount-rate facts and helps explain lower growth and higher markups, while weaker catch-up and changes in the R&D technology explain why aggregate research productivity falls. The calibrated transition also reproduces the present-value decomposition of firm valuations in Cho et al. (2026), both in the aggregate and within firms: higher valuations reflect higher expected markups and a lower financial cost of capital, partly offset by higher expected R&D. Thus, the patterns they document with a reduced-form present-

[6]For evidence of weaker catch-up and a more scale-intensive R&D technology associated with specialization and team science, see Olmstead-Rumsey (2025) and Kim et al. (2026).

value identity emerge from our creative-destruction framework with an endogenous discount-rate gap.

Related literature. Our paper contributes to the Schumpeterian literature on innovation, market competition, and growth, which has recently investigated rising advantages for market leaders (Aghion et al. 2023; Akcigit and Ates 2023; De Ridder 2024). In these models, the discount rate governing firms' innovation decisions coincides with the financial cost of capital. We endogenize a gap between these rates and study how it interacts with creative destruction. This links Schumpeterian models to evidence that firms' hurdle rates exceed their cost of capital (Poterba and Summers 1995; Gormsen and Huber 2025; Barry et al. 2026). The model's cross-sectional implications for these gaps are consistent with the data.

Our paper is complementary to Liu et al. (2022), who show that, when follower catch-up is gradual, a lower long-term interest rate can strengthen leaders' innovation incentives relative to followers', raising concentration and reducing productivity growth. They study a single discount rate, common to financial valuation and firm R&D decisions, moved by changes on the consumer side that push consumers toward saving more and consuming less. We study an endogenous gap: the financial cost of capital can fall while firms' required return on innovation remains high or rises, because innovation exposes decision-makers to undiversifiable risk and because innovation profits are imperfectly pledgeable. This difference yields distinct cross-sectional predictions, which we take to the data. Their insight that the effects of discount rates depend on the creative-destruction process also motivates our emphasis on moments informative about firm dynamics, markups, leadership persistence, and reallocation.

Our paper also connects to macro-finance accounts of weak investment and high valuations; Farhi and Gourio (2018) point to market power, unmeasured intangibles, and risk premia as key drivers. We endogenize productivity growth, market power, and the discount-rate gap in a creative-destruction model. Relatedly, in Terry (2023), short-term pressure to meet profit targets disciplines managers who derive private benefits from R&D; short-termism can raise firm value while reducing R&D and growth. In our model, firm profits rise because a higher hurdle rate weakens creative destruction, making market leadership durable. In research on finance and growth, credit constraints can reduce growth by shifting investment toward shorter-horizon projects (Aghion et al. 2010), and financial frictions can distort the allocation of investment between capital and innovation (Ottonello and Winberry 2024). Our focus is the discount-rate gap and its consequences for innovation and market competition.

Our paper brings idiosyncratic capital-income risk (Angeletos and Panousi 2011; Di Tella and Hall 2021) into a Schumpeterian environment. The assumption that innovation investment exposes decision-makers to undiversifiable risk is natural for entrepreneurs, venture capitalists, and insiders with high ownership or high-powered incentives. Fat-tailed innovation outcomes and agency frictions that limit diversification can leave capital allocators exposed to idiosyncratic risk (Gârleanu and Panageas 2024). Empirically, Krieger et al. (2021) find that even large innovative firms may behave as if they are averse to idiosyncratic risk, and Panousi and Papanikolaou (2012) show that idiosyncratic risk reduces investment more when managers own larger equity stakes. The concern is longstanding: Schumpeter (1934) emphasized uncertainty in entrepreneurial innovation, and Scherer (1980) argued that R&D risk is borne by individual managers and investors, not by impersonal organizations.

1. Model

Time is continuous, indexed by $t \geq 0$. A continuum of intermediate-goods industries is indexed by $j \in [0, 1]$. Two firms compete in each industry and finance R&D subject to a limited pledgeability constraint. Risk-averse entrepreneurs are infinitely lived, own firms, can trade a risk-free bond, and are indexed by $i \in [0, 1]$. Entrepreneurs are exposed to idiosyncratic shocks that they can only partly diversify.

1.1 Preferences and goods markets

We denote the consumption of an individual entrepreneur by c_t^i. Entrepreneurs have Duffie and Epstein (1992) stochastic differential utility with

$$J_t^i = \lim_{\tau \to \infty} E_t \left[\int_t^\tau \Upsilon(c_s^i, J_s^i)\, ds \right], \tag{1}$$

where $\Upsilon(c, J)$ is the utility aggregator[7]

$$\Upsilon(c, J) = \frac{\rho}{1 - \epsilon^{-1}} \left\{ \frac{c^{1-\epsilon^{-1}}}{[(1-\nu)J]^{\frac{\nu - \epsilon^{-1}}{1-\nu}}} - (1-\nu)J \right\}. \tag{2}$$

Relative risk aversion is ν, the elasticity of intertemporal substitution (EIS) is ϵ, and the time-preference rate is ρ. There is a continuum of workers who supply labor inelastically and consume their wage income. The assumption of hand-to-mouth workers keeps the asset-pricing side of the model tractable and lets us characterize hurdle rates and the financial cost of capital. The mass of workers is normalized to one, and each worker has one unit of labor endowment.

[7]These preferences are a continuous-time analog of Epstein and Zin (1989) preferences. The limiting cases $\nu = 1$ and $\epsilon = 1$ are obtained by continuity. For $\nu > 1$, continuation utility is negative, so $(1 - \nu)J > 0$.

Let Y_t denote output of the final good, produced according to

$$Y_t = \exp\left\{\int_0^1 \ln\left[y_{1,j,t}^{\zeta} + y_{2,j,t}^{\zeta}\right]^{1/\zeta} dj\right\}, \tag{3}$$

where $y_{z,j,t}$ is the quantity produced by firm $z \in \{1,2\}$ in industry j. The parameter $\zeta \in (0,1)$ governs within-industry substitutability, with higher ζ implying closer substitutes. Thus, the final-good technology is Cobb–Douglas across industries and has constant elasticity of substitution (CES) within each industry. The final good is the numeraire and is sold in a perfectly competitive market.

Intermediate-goods output is $y_{z,j,t} = q_{z,j,t} l_{z,j,t}$, where $q_{z,j,t}$ is productivity and $l_{z,j,t}$ is labor hired at wage w_t. Let $p_{z,j,t}$ denote firm z's price. Cobb–Douglas aggregation implies equal expenditure across industries, $p_{1,j,t}y_{1,j,t} + p_{2,j,t}y_{2,j,t} = Y_t$, while within-industry demand satisfies $\frac{p_{1,j,t}}{p_{2,j,t}} = \left(\frac{y_{1,j,t}}{y_{2,j,t}}\right)^{\zeta-1}$. Firms compete in prices, à la Bertrand.

1.2 Innovation, technology gaps, and profits

Firms in each industry are ordered on a quality ladder and invest in R&D to move upward. Each rung represents a proportional productivity improvement of scale $\lambda > 1$. Let $s \in \{0, 1, \ldots, \bar{s}\} \equiv S^+$ denote the number of rungs separating leader and laggard, where $\bar{s}$ is the maximal gap. A firm's technology position is denoted by $\sigma \in \{-\bar{s}, \ldots, \bar{s}\} \equiv S$. A firm with $\sigma > 0$ is a leader, a firm with $\sigma < 0$ is a laggard, and $\sigma = 0$ denotes a tied industry. We drop the time t and industry j indexes when no confusion arises.

A firm in position σ innovates at rate $x_t(\sigma)$ by hiring $G(x_t(\sigma)) = (\frac{x_t(\sigma)}{B})^{\frac{1}{\gamma}}$ R&D workers, where B is an R&D cost scalar and $\gamma \in (0,1)$ governs the convexity of R&D costs. Leaders innovate to pull further ahead, while laggards innovate to catch up. An innovating leader advances one rung. A successful laggard innovation has two possible outcomes. With probability $1-\phi$, the laggard advances one rung. With probability ϕ, it catches up to the leader.[8]

The share of industries with technology gap s is denoted by $\mu_t(s)$. For $s > 0$,

$$\dot{\mu}_t(s) = \underbrace{\mu_t(s+1)(1-\phi)x_t(-(s+1)) + \mu_t(s-1)x_t(s-1)(1+\mathbb{1}_{s=1})}_{\text{Inflows to gap } s} - \underbrace{(x_t(s)+x_t(-s))\mu_t(s)}_{\text{Outflows from gap } s}. \tag{4}$$

Inflows come from incremental laggard innovation in industries with gap $s+1$ and from leader innovation in industries with gap $s-1$. Outflows come from leader or laggard innovation. The indicator function for $s = 1$, or $\mathbb{1}_{s=1}$, takes into account that a tied industry shifts to a one-rung gap when either of the two tied firms innovates.[9]

In an industry with gap $s > 0$, the leader's productivity is λ^s times the laggard's. Bertrand competition determines the leader-to-laggard relative price m_s, which is defined implicitly by $m_s = \lambda^{-s}\left(\frac{1+(1-\zeta)m_s^{-\frac{\zeta}{1-\zeta}}}{1+(1-\zeta)m_s^{\frac{\zeta}{1-\zeta}}}\right)$ and is decreasing in s. Because total expenditure in each industry equals Y_t, operating profits can be written as $\Pi_t(\sigma) = Y_t\pi(\sigma)$, where

$$\pi(s) = \frac{1-\zeta}{m_s^{\frac{\zeta}{1-\zeta}}+1-\zeta}, \qquad \pi(-s) = \frac{1-\zeta}{m_s^{-\frac{\zeta}{1-\zeta}}+1-\zeta}, \qquad \pi(0) = \frac{1-\zeta}{2-\zeta}. \tag{5}$$

Scaled profits $\pi(\sigma)$ and the associated markups are increasing in the technology position σ. Internet Appendix C.1 derives expressions for profits, markups, and labor demand.[10]

1.3 Firm value, financing, and R&D

Each firm's R&D decisions are inherently strategic and take into account its competitor's strategy. Let $\{x_t^c(\sigma)\}_{\sigma\in S}$ denote the innovation strategy of a firm's competitor, where $x_t^c(\sigma)$

[8] A leader with an $\bar{s}$-rung advantage has achieved the maximum feasible lead and does not undertake R&D. In the calibration, $\bar{s}$ is set large enough that there is negligible mass of industries with gap $\bar{s}$.

[9] For completeness, $\mu_t(0)$ is determined by the identity $\sum_{s=0}^{\bar{s}}\mu_t(s) = 1$. By convention, $\mu_t(\bar{s}+1) = 0$, so that Eq. (4) applies when $s = \bar{s}$.

[10] As ζ goes to 1, these expressions converge to their values with perfect substitution ($\zeta = 1$), which implies limit pricing (leaders set the price to the marginal cost of the laggard).

is the innovation rate of a competitor in position σ. A firm in position σ therefore faces competitor innovation rate $x_t^c(-\sigma)$.

For a firm in position σ, the expected value of firm profits, discounted at entrepreneurs' required return r_t, is denoted by $V_t(\sigma)$. Let $\Delta V_t(\sigma)$ denote the expected capital gain from the firm's own innovation:

$$\Delta V_t(\sigma) = \begin{cases} V_t(\sigma+1) - V_t(\sigma) & \text{if } 0 \leq \sigma < \bar{s}, \\ (1-\phi)V_t(\sigma+1) + \phi V_t(0) - V_t(\sigma) & \text{if } \sigma < 0. \end{cases} \tag{6}$$

This expression reflects that innovating leaders and tied firms advance one rung, while innovating laggards have the possibility of catching up. Let $\Delta^c V_t(\sigma)$ denote the (typically negative) expected capital gain when the firm's competitor innovates:

$$\Delta^c V_t(\sigma) = \begin{cases} (1-\phi)V_t(\sigma-1) + \phi V_t(0) - V_t(\sigma) & \text{if } 0 < \sigma \leq \bar{s}, \\ V_t(\sigma-1) - V_t(\sigma) & \text{if } -\bar{s} < \sigma \leq 0. \end{cases} \tag{7}$$

For a leader, competitor innovation reduces the leader's technology position by one rung with probability $1-\phi$, or to zero with probability ϕ.[11]

The value of the firm satisfies

$$r_t V_t(\sigma) - \dot{V}_t(\sigma) = \max_{x_t(\sigma)} \left\{ \Pi_t(\sigma) - w_t G(x_t(\sigma)) + x_t(\sigma)\Delta V_t(\sigma) + x_t^c(-\sigma)\Delta^c V_t(\sigma) \right\}, \tag{8}$$

subject to the pledgeability constraint

$$w_t G(x_t(\sigma)) \leq \alpha V_t(\sigma), \tag{9}$$

where α is the pledgeable share of firm value. The firm's innovation rate is then

$$x_t(\sigma) = \min\left\{ (G')^{-1}\left(\frac{\Delta V_t(\sigma)}{w_t}\right), G^{-1}\left(\frac{\alpha V_t(\sigma)}{w_t}\right) \right\}, \qquad \sigma < \bar{s}. \tag{10}$$

The first term is the unconstrained optimum. The second is the maximum innovation rate consistent with the borrowing limit implied by limited pledgeability. When the

[11]A firm in position $\sigma = \bar{s}$ does not innovate, and so we set $\Delta V_t(\bar{s}) = 0$ and $\Delta^c V_t(-\bar{s}) = 0$.

financing constraint binds, the firm innovates less than it would if unconstrained and behaves as if it uses a higher discount rate in evaluating R&D.

1.4 Entrepreneurs, uninsured risk, and the financial cost of capital

Entrepreneur i's net worth $n_t^i = k_t^i + b_t^i$ is the sum of the holdings in innovative firms, k_t^i, and in the risk-free bond, b_t^i. Holdings in innovative firms earn expected return r_t but expose the entrepreneur to uninsured idiosyncratic risk:

$$dk_t^i = r_t k_t^i dt + \sigma_{A,t} k_t^i dZ_t^i, \tag{11}$$

where Z_t^i is a standard Brownian motion, independent across entrepreneurs. The deterministic sequence $\sigma_{A,t}$ captures the level of uninsured idiosyncratic risk. The shock dZ_t^i captures entrepreneur-specific variation in the realized return on innovative investment after any private contracting. As in the Bewley tradition, we exogenously impose limits on risk sharing: the assumption $\sigma_{A,t} > 0$ reflects incentive-provision motives or contracting frictions that limit risk sharing. Equivalently, we could set $\sigma_{A,t} = \Lambda_t \tilde{\sigma}_{A,t}$, where $\tilde{\sigma}_{A,t} > 0$ is the raw idiosyncratic risk and $\Lambda_t > 0$ captures the severity of the friction. Internet Appendix C.3 provides a foundation for this setup by allowing entrepreneurs to sell equity on only a fraction of their profits.[12] With consumption c_t^i, net worth evolves according to

$$dn_t^i = \left(r_t k_t^i + r_t^f b_t^i - c_t^i \right) dt + \sigma_{A,t} k_t^i dZ_t^i, \tag{12}$$

where the risk-free bond earns return r_t^f. Because there is no aggregate risk, the risk-free rate also prices diversified claims on innovative firms. We therefore refer to r_t^f interchangeably as the risk-free rate and the financial cost of capital.

Given prices, the entrepreneur chooses consumption and the portfolio share in in-

[12] Internet Appendix C.4 describes a simple foundation of entrepreneurs' firm-level portfolios that does not alter the characterization of creative-destruction dynamics in this section.

novative firms.[13] Because the problem is homothetic in net worth, optimal policies are linear, with consumption $c_t^i = c_t n_t^i$, investment in innovative firms $k_t^i = \xi_t n_t^i$, and bond holdings $b_t^i = (1-\xi_t)n_t^i$, where ξ_t is the portfolio share in innovative firms. The instantaneous risk-adjusted return on the entrepreneur's portfolio is

$$\widehat{r}_t(\xi) = r_t^f + \xi(r_t - r_t^f) - \frac{1}{2}\nu\xi^2\sigma_{A,t}^2. \tag{13}$$

The last term is the adjustment for uninsurable risk. The portfolio share maximizes the risk-adjusted return in Eq. (13), subject to the natural borrowing constraint. The consumption-wealth ratio evolves according to

$$\frac{\dot{c}_t}{c_t} = c_t + (\epsilon - 1)\widehat{r}_t(\xi_t) - \epsilon\rho. \tag{14}$$

Internet Appendix C.2 summarizes and proves these results.

1.5 Equilibrium and balanced growth

Because idiosyncratic uncertainty washes out in the aggregate, equilibrium aggregate quantities are deterministic. Let C_t^E, K_t, and $\mathcal{B}_t$ denote the aggregate levels of entrepreneurial consumption, holdings in innovative firms, and bond holdings at date t; that is, the cross-sectional averages of c_t^i, k_t^i, and b_t^i. An equilibrium consists of prices $\{w_t, r_t, r_t^f\}_{t\geq 0}$, entrepreneurial plans $\{c_t^i, k_t^i, b_t^i\}_{t\geq 0}$ for $i \in [0,1]$, firm policies and values $\{x_{f,t}, p_{f,t}, l_{f,t}, V_{f,t}\}_{t\geq 0}$ for $f \in [0,1] \times \{1,2\}$, aggregate quantities $\{C_t^E, K_t, \mathcal{B}_t, Y_t\}_{t\geq 0}$, and the distribution of technology gaps $\{\mu_t(s)\}_{t\geq 0}$ for $s \in S^+$ such that entrepreneurs and firms optimize, competitor strategies are symmetric ($x_t^c(\sigma) = x_t(\sigma)$ for $\sigma \in S$), the gap distribution evolves according to Eq. (4), and the goods, labor, firm-ownership, and bond markets clear, with bonds in zero net supply ($\mathcal{B}_t = 0$). Internet Appendix C.5 states the

[13] Consumption and investment in innovative firms are nonnegative. Bond positions satisfy a natural borrowing constraint that keeps net worth nonnegative.

full equilibrium system and discusses the equivalence of the number of restrictions and endogenous variables.

Along a balanced growth path (BGP), idiosyncratic risk is constant, $\sigma_{A,t} = \sigma_A$; output, entrepreneurial consumption, and worker consumption grow at a constant rate g; the distribution of technology gaps is stationary; and the required return r is constant, with $r > g$ so scaled firm values are finite. The labor share $\omega = w_t/Y_t$ is constant, and so is the scaled firm value $v(\sigma) = V_t(\sigma)/Y_t$. The BGP gap distribution satisfies Eq. (4) with $\dot{\mu}(s) = 0$. Aggregate productivity growth is

$$g = \ln \lambda \sum_{s \in S^+} \mu(s)(1 + \mathbb{1}_{s=0})x(s). \tag{15}$$

Labor market clearing requires production labor and R&D labor to sum to one:

$$\sum_{s \in S^+} \mu(s) \left[G(x(s)) + G(x(-s)) + l(s) + l(-s)\right] = 1, \tag{16}$$

where $l(\sigma)$ denotes production labor for a firm in position σ.

Along a BGP, scaled expected capital gains from innovation are $\Delta v(\sigma) = \Delta V_t(\sigma)/Y_t$ and $\Delta^c v(\sigma) = \Delta^c V_t(\sigma)/Y_t$. The firm's innovation condition can be written as if the firm discounts innovation payoffs at a position-specific hurdle rate $r^k(\sigma)$:

$$G'(x(\sigma))\omega = \frac{r - g}{r^k(\sigma) - g} \Delta v(\sigma). \tag{17}$$

Comparing Eq. (17) with the innovation condition in Eq. (10), if the financing constraint does not bind for position σ, then $r^k(\sigma) = r$. If it binds, then $r^k(\sigma) > r$. The model therefore allows hurdle rates to differ across firms, even within the same aggregate environment, because expected innovation gains and financing constraints differ across positions on the technology ladder.

2. The endogenous discount-rate gap

This section studies the gap between the discount rates reflected in firms' R&D choices and the rate used in financial markets to value firms' cash flows. We show that this gap is positive and can vary across firms. We characterize the two components of this gap—a risk premium for uninsured idiosyncratic risk and a financing wedge generated by limited pledgeability. We also show that greater exposure to undiversifiable idiosyncratic risk raises the discount-rate gap and lowers the cost of capital, with the average hurdle rate moving less (hurdle rates are sticky).

For a firm in position σ, the discount-rate gap is

$$\kappa(\sigma) \equiv r^k(\sigma) - r^f. \tag{18}$$

A positive $\kappa(\sigma)$ means that the marginal R&D investment earns a return above the financial cost of capital. Equivalently, a marginal R&D investment with a return between r^f and $r^k(\sigma)$ has positive value when discounted by financial markets but is not pursued by the firm. The aggregate hurdle rate and the aggregate gap are the cross-sectional averages:[14]

$$r^k = \sum_{s \in S^+} \frac{1}{2}\mu(s)\left(r^k(s) + r^k(-s)\right), \qquad \kappa = \sum_{s \in S^+} \frac{1}{2}\mu(s)\left(\kappa(s) + \kappa(-s)\right). \tag{19}$$

The firm-level gap has two components:

$$\kappa(\sigma) = \underbrace{r - r^f}_{\text{idiosyncratic-risk premium}} + \underbrace{r^k(\sigma) - r}_{\text{financing wedge}}. \tag{20}$$

The first term, which is common across firms, reflects the compensation that entrepreneurs require for bearing uninsured idiosyncratic risk when they own and operate innovative

[14]The $\frac{1}{2}$ terms in Eq. (19) reflect that we sum discount rates or gaps for the two firms in each industry; multiplying by $\frac{1}{2}$ converts that sum into the cross-sectional mean.

firms. The second term, which is specific to a firm's technology position, reflects the R&D financing constraint. In equilibrium, risk and the financing constraint interact because risk affects firm values, financing constraints, innovation rates, and the distribution of technology gaps. Nonetheless, the usefulness of the decomposition in Eq. (20) is highlighted by the next result, which provides a closed-form characterization of the idiosyncratic-risk premium and connects the financing wedge to the position-specific tightness of the financing constraints. To do so, we let $\chi(\sigma) = \frac{\Delta v(\sigma)}{\omega G'(x(\sigma))} - 1 \geq 0$ denote the Lagrangian wedge associated with the financing constraint of a firm in position $\sigma < \bar{s}$.[15]

Proposition 1 (Endogenous discount-rate gap) *Along a balanced growth path, the idiosyncratic-risk premium and firm-specific financing wedge are given by*

$$r - r^f = \nu\sigma_A^2 \qquad \textit{and} \qquad r^k(\sigma) - r = (r - g)\chi(\sigma). \tag{21}$$

Therefore, the firm-specific discount-rate gap is $\kappa(\sigma) = \nu\sigma_A^2 + (r-g)\chi(\sigma) \geq 0$. *The aggregate discount-rate gap is*

$$\kappa = \nu\sigma_A^2 + (r - g)\sum_{s\in S^+}\frac{1}{2}\mu(s)\left(\chi(s) + \chi(-s)\right) \geq 0. \tag{22}$$

These inequalities are strict if $\sigma_A > 0$.

Proposition 1 identifies the two forces behind the endogenous discount-rate gap. The term $\nu\sigma_A^2$ is the idiosyncratic-risk premium. This innovator's risk premium is positive when entrepreneurs face uninsured idiosyncratic risk. The term $(r - g)\chi(\sigma)$ is the financing wedge. If the R&D financing constraint binds, a firm behaves as if it uses a discount rate above entrepreneurs' required return. Both terms are absent in the standard Schumpeterian framework with complete risk sharing and frictionless R&D finance. Because

[15] We set $\chi(\bar{s}) = 0$, as firms with $\sigma = \bar{s}$ already have the maximum advantage.

$\chi(\sigma)$ varies with a firm's position on the technology ladder, the model also generates cross-sectional variation in discount-rate gaps.

The proposition also clarifies how idiosyncratic risk and financing constraints can amplify each other. A rise in idiosyncratic risk directly raises the risk premium $r - r^f$. It can also tighten financing constraints by reducing the value of pledgeable claims to firm profits, shifting the distribution of technology gaps, and changing $r - g$. These general-equilibrium channels affect the financing-wedge term and, as we show below, matter for the rise in the aggregate hurdle rate. Note also that, in standard Schumpeterian models, with log utility, $r - g$ equals ρ and hence is completely pinned down by the time-preference rate. In our quantitative analysis, a rise in idiosyncratic risk implies that r rises while g falls, widening the financing wedge component of the discount-rate gap (even if the tightness of financing constraints is held constant). The next result studies how a wider gap is split between a lower financial cost of capital and relatively sticky hurdle rates.[16]

Proposition 2 (A depressed cost of capital and sticky required returns) *Along a balanced growth path, the financial cost of capital and the entrepreneurial return satisfy*

$$r^f = \rho + \frac{g}{\epsilon} - \frac{1}{2}\left(1 + \frac{1}{\epsilon}\right)\nu\sigma_A^2, \quad \textit{and} \quad r = \rho + \frac{g}{\epsilon} + \frac{1}{2}\left(1 - \frac{1}{\epsilon}\right)\nu\sigma_A^2. \tag{23}$$

Hence a rise in idiosyncratic risk depresses the growth-adjusted financial cost of capital $r^f - g/\epsilon$. *Moreover, the entrepreneurial return—a key component of the hurdle rate* r^k*—moves less with idiosyncratic risk than the financial cost of capital:*

$$\left|\frac{\partial}{\partial\sigma_A^2}\left(r - \frac{1}{\epsilon}g\right)\right| = \frac{1}{2}\left|1 - \frac{1}{\epsilon}\right|\nu < \frac{1}{2}\left(1 + \frac{1}{\epsilon}\right)\nu = \left|\frac{\partial}{\partial\sigma_A^2}\left(r^f - \frac{1}{\epsilon}g\right)\right|. \tag{24}$$

[16] Some results in Proposition 2 are stated in terms of growth-adjusted rates. Because productivity growth g is endogenous, the total response of the raw rates r and r^f also includes the common growth term g/ϵ, which we quantify below.

Risk-free bonds and diversified claims on firm cash flows are not exposed to idiosyncratic risk. When risk rises, entrepreneurs increase their precautionary demand for these assets, depressing the financial cost of capital. At the same time, entrepreneurs require a larger premium to own innovative capital. The change in the hurdle rate reflects the lower cost of capital offset by the higher premium. Greater idiosyncratic risk lowers the cost of capital with coefficient proportional to $1 + 1/\epsilon$, while the entrepreneurial return that underlies hurdle rates moves less, with coefficient proportional to $1 - 1/\epsilon$. Although decision-makers in the model are forward-looking, equilibrium behavior is thus observationally as if firms do not pass the decline in the financial cost of capital through to their hurdle rates. Note that the positive gap $\kappa > 0$ and the asymmetry above (the relative stickiness of entrepreneurs' required return) hold for any EIS. With $\epsilon > 1$, the case emphasized in our quantitative analysis, the substitution effect from the lower risk-adjusted return dominates the income effect, so the growth-adjusted entrepreneurial return rises with idiosyncratic risk.

For computation, the balanced-growth problem can be written in a form close to a standard Schumpeterian model. Combining Eq. (8) with Eq. (23), the scaled firm value satisfies

$$[\bar{\rho} - (1 - \frac{1}{\epsilon})g]v(\sigma) = \pi(\sigma) - G(x(\sigma))\omega + x(\sigma)\Delta v(\sigma) + x(-\sigma)\Delta^c v(\sigma), \tag{25}$$

subject to the financing constraint $\omega G(x(\sigma)) \leq \alpha v(\sigma)$, where $\bar{\rho} \equiv \rho + \frac{1}{2}\left(1 - \frac{1}{\epsilon}\right)\nu\sigma_A^2$. Thus, the aggregate allocation (innovation rates, productivity growth, the technology-gap distribution) can be obtained by solving the model's Schumpeterian block with the adjusted time-preference rate $\bar{\rho}$; Lemma IA.4 in Internet Appendix C.2 states this equivalence formally. However, equilibrium discount rates behave differently from a standard model with this adjusted time-preference rate: there is a positive wedge $r^k - r^f$, a depressed financial cost of capital, and a relatively sticky hurdle rate.

Even with this representation, solving for the aggregate allocation differs from solving for the allocation in a standard Schumpeterian model because the financing constraint must hold and the EIS need not equal one, which makes equilibrium discounting (of growing firm profits) depend on aggregate growth (Eqs. (23) and (25)). We iterate on conjectures for the growth rate g and labor share ω. Given (g, ω), we solve the scaled value function by uniformization (Acemoglu and Akcigit 2012) and value-function iteration, imposing the financing constraint at each step. The implied innovation rates determine the stationary distribution of technology gaps, aggregate growth, and labor demand. We update (g, ω) until the growth-rate and labor-market-clearing conditions, Eqs. (15) and (16), are satisfied.

Aggregate profitability and financial-market values. One measure of profitability is the aggregate earnings-to-sales ratio, equal to aggregate firm earnings (operating profits net of R&D costs) divided by aggregate sales. This ratio is $1 - \omega$, because ω is both the scaled wage and the labor share. Aggregate price-to-sales can then be obtained using the aggregate P/E ratio. The distinction between internal and financial discount rates matters for valuation. Although the hurdle rate governs R&D choices, diversified claims on firms are priced using r^f and take firm innovation decisions as given.

Along a balanced growth path, the financial-market value of a firm in position σ, scaled by output and denoted $v^f(\sigma)$, satisfies:[17]

$$(r^f - g)v^f(\sigma) = \pi(\sigma) - G(x(\sigma))\omega + x(\sigma)\Delta v^f(\sigma) + x(-\sigma)\Delta^c v^f(\sigma). \tag{26}$$

[17]Here, $\Delta v^f(\sigma)$ and $\Delta^c v^f(\sigma)$ are defined analogously to $\Delta v(\sigma)$ and $\Delta^c v(\sigma)$, replacing v by v^f.

A claim to aggregate firm earnings is inherently diversified; the Gordon formula gives

$$\text{P/E} = \frac{1}{r^f - g} = \frac{1}{\rho + \left(\frac{1}{\epsilon} - 1\right) g - \frac{1}{2}\left(1 + \frac{1}{\epsilon}\right) \nu \sigma_A^2}. \tag{27}$$

A rise in idiosyncratic risk lowers the denominator in Eq. (27), reflecting the precautionary-savings force that depresses the financial cost of capital; it also affects the P/E ratio through its effect on the growth rate.

3. Calibration

We next calibrate the parameters that determine the discount-rate gap and govern the creative-destruction process through which the wedge affects growth and market power. We therefore target the discount-rate gap, the financial cost of capital, productivity growth, and aggregate and cross-sectional moments related to innovation, firm profitability, and firm dynamics. We choose the seven parameters $(\phi, \lambda, B, \alpha, \sigma_A, \rho, \zeta)$ jointly by minimizing a minimum-distance criterion. Although each parameter affects all targeted moments to some degree, we begin by describing how specific targets are informative about particular parameters.

3.1 Targeted moments and identification

To compute model moments for each candidate parameter vector, we solve the model at a monthly frequency, simulate a panel of firms, and aggregate to an annual frequency. We minimize

$$\min_{\phi,\lambda,B,\alpha,\sigma_A,\rho,\zeta} \sum_{m=1}^{M} \text{Weight}_m \left(\frac{|\text{model}(m) - \text{data}(m)|}{\frac{1}{2}|\text{model}(m)| + \frac{1}{2}|\text{data}(m)|} \right). \tag{28}$$

Each category of moments receives the same total weight. For example, the two R&D-to-sales moments together receive the same weight as productivity growth. Throughout the paper, discount rates, productivity growth, and the parameters $(\sigma_A, \rho, B, \alpha)$ are reported in annualized terms.

To put discipline on time preference, ρ, and idiosyncratic risk, σ_A, we target the financial cost of capital and the discount-rate gap. Discount-rate targets are from Gormsen and Huber (2025). We use a real financial cost of capital of 5.20% and an aggregate discount-rate gap of $\kappa = 2.64\%$.[18] The implied hurdle rate is $r^f + \kappa = 7.84\%$.

The two discount-rate targets separate ordinary time preference from uninsured idiosyncratic risk because the two parameters move the financial cost of capital and the gap differently. As Eq. (23) shows, a higher time-preference rate raises entrepreneurs' required return and the financial cost of capital in parallel. The idiosyncratic-risk premium $r - r^f = \nu\sigma_A^2$ in Proposition 1 does not depend on ρ, so ρ shifts the level of discount rates while affecting the gap only through general-equilibrium movements in the financing wedge. By contrast, higher idiosyncratic risk moves the two rates apart: it raises the compensation entrepreneurs require for bearing undiversifiable risk while lowering the financial cost of capital through precautionary saving. The level of the financial cost of capital is therefore informative about ρ, and the discount-rate gap is informative about σ_A.

To pin down α, we use its distinct imprint on the cross section of R&D-to-sales and innovation output.[19] Lower α reduces average R&D-to-sales and average innovation

[18] The aggregate wedge is obtained from Gormsen and Huber (2025) using discount rates for firms that fully account for overhead costs in their cash-flow analyses, as described in Internet Appendix A. The targeted cost of capital is the average perceived nominal cost of capital, 8.4%, from Gormsen and Huber (2025), less the 2.2% average annual inflation rate over their sample period and a further 1.0% adjustment for non-TFP sources of growth absent from our model, such as physical-capital deepening.

[19] The discount-rate gap also helps discipline pledgeability α. A lower value of α makes innovation profits less pledgeable, tightens financing constraints, and raises the financing-wedge component of firms' required returns. This channel has little direct effect on the financial cost of capital; its effect on r^f operates mainly through equilibrium growth.

output by tightening the financing constraint for a broad set of firms. However, lower α raises R&D-to-sales for top-profit firms, which have high firm values, low capital gains from a marginal innovation, and hence tend to be unconstrained. These firms raise R&D because weaker innovation by financially constrained competitors reduces the threat that their innovations will be eroded by catch-up. Relatedly, lower α reduces average innovation output but increases the 90$^{\text{th}}$ percentile of innovation output.

Average R&D-to-sales ratios for all firms and for firms in the top profit quintile are, respectively, 4.37% and 2.65%, computed from CRSP/Compustat data. Innovation output is from Kogan et al. (2017), who measure the economic value of innovations using stock-market reactions to patent grants. Following their normalization, firm-level innovation output is the value of new innovations scaled by firm size. We target a mean of 6.77% and a 90$^{\text{th}}$ percentile of 19.54%.

To put discipline on the innovation step size λ, we target productivity growth. The target is 1.03%, from Fernald et al. (2017). To discipline the CES parameter ζ, we target the mean net markup. The target is 19.41%, based on Hall (2018). Given the innovation step size and the gap distribution, the mean net markup pins down ζ, which governs within-industry substitutability. To put discipline on the R&D cost scalar B, we target average R&D-to-sales for all firms and firms in the top profit quintile. Higher B lowers marginal R&D cost, raising both R&D spending and innovation output.[20] To help discipline laggard catch-up ϕ, we target the innovation-output distribution and profit volatility: catch-up innovations are especially valuable for laggards that advance multiple rungs, raising the right tail of innovation output, while the displacement risk that catch-up creates drives

[20] Higher B increases average R&D-to-sales for all firms and firms in the top profit quintile, whereas lower α reduces average R&D-to-sales for all firms while raising R&D-to-sales for top-profit firms.

profit volatility among market leaders. Profit volatility is the standard deviation of profit growth, computed for all firms and for firms in the top profit quintile in the prior year.[21] Internet Appendix D describes the data construction.[22]

3.2 Externally calibrated parameters and model fit

We set the R&D curvature parameter to $\gamma = 0.5$, as in Acemoglu et al. (2018) and De Ridder (2024). The maximum technology gap is $\bar{s} = 100$, and we normalize relative risk aversion to one ($\nu = 1$); the jointly calibrated idiosyncratic risk σ_A and time-preference rate ρ then match the financial cost of capital and the discount-rate gap.[23] We set $\epsilon = 2$ for the elasticity of intertemporal substitution.[24] As Section 2 shows, an EIS above one is needed for a rise in uninsured idiosyncratic risk to generate the sign pattern emphasized in the introduction and studied below: a higher hurdle rate together with a lower financial cost of capital. Internet Appendix E discusses sensitivity to externally calibrated parameters including the EIS.

Table 1 reports the calibrated parameters and model fit. The estimated step size is

[21] Profit growth is measured as operating profits in a given year divided by operating profits in the prior year.

[22] We calculate moments in the model using a simulation of 1,100 industries for 60 years. The number of firm-years used to calculate moments in the model approximates the number of firm-years in the data underlying our profit-volatility and R&D-to-sales moments.

[23] Because there is no aggregate risk, ν enters the model only through the idiosyncratic-risk premium, $\nu\sigma_A^2$. Thus, an alternative value of ν would imply the same equilibrium wedge and allocations if σ_A^2 were also rescaled. This normalization is analogous to the quantification of models where mean-variance investors face asset-supply shocks (e.g., Greenwood et al. 2018), in which risk premia depend on the quantity of risk relative to aggregate risk tolerance. These papers typically normalize the risk aversion to 1 and then choose the volatility of supply shocks to match risk premia, or normalize the volatility of supply shocks to 1 and then select relative risk aversion. Our paper's approach is akin to the former.

[24] Estimates of the EIS using aggregate data (e.g., Hall 1988) generally find low values, but in simulated data from our model, the analogous Hall-style regression of entrepreneurial consumption growth on the risk-free rate gives a coefficient of 0.11, consistent with estimates from aggregate data and well below the structural EIS in the model. Using variation in after-tax capital-income returns, Gruber (2013) estimates an EIS around 2. Long-run-risk asset-pricing estimates and calibrations also have EIS values above one, and often near 2, consistent with the implication that higher uncertainty lowers valuations (Bansal et al. 2016). Internet Appendix E.3 discusses these issues.

Table 1: **Calibrated parameters and targeted moments**

Parameter values			Moments used in calibration		
Parameter	Description	Value	Description	Model	Data
ϕ	Laggard catch-up	0.51	Productivity growth	1.08%	1.03%
λ	Innovation step size	1.05	Mean net markup	19.10%	19.41%
B	R&D cost scalar	1.22	Cost of capital	5.24%	5.20%
α	Pledgeability	3.81%	Discount-rate gap, κ	2.64%	2.64%
σ_A	Idiosyncratic risk	13.85%	Profit volatility		
ρ	Time preference	6.14%	All firms	35.60%	35.75%
ζ	CES parameter	0.97	Top profit quintile	16.59%	19.45%
			Average R&D-to-sales		
			All firms	4.40%	4.37%
			Top profit quintile	3.30%	2.65%
			Innovation output		
			Mean	4.44%	6.77%
			90^{th} percentile	19.03%	19.54%

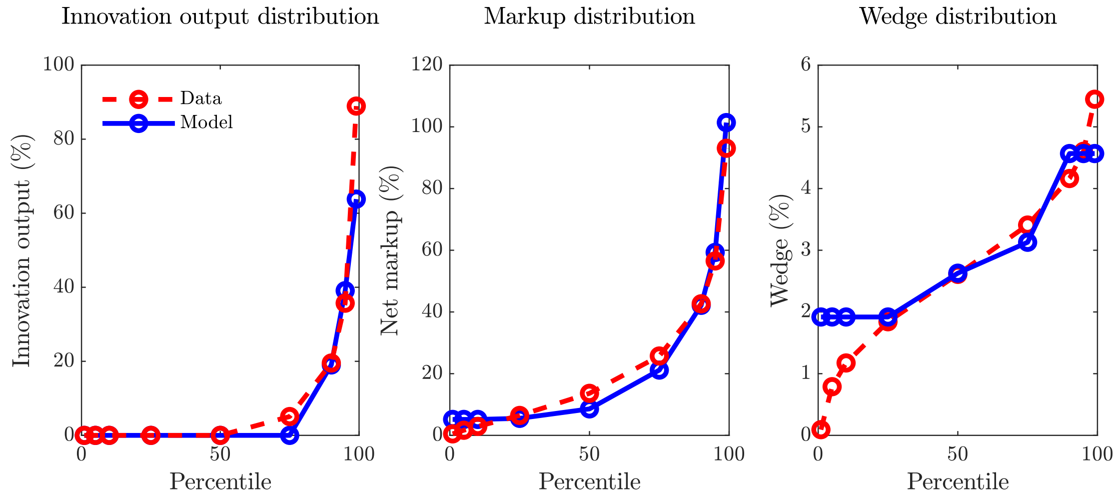
Discount rates, productivity growth, and the parameter values for B, α, σ_A, and ρ are reported in annual terms. Profit volatility is computed from annual firm-level profit growth and is not annualized. R&D-to-sales targets are averages.

$\lambda = 1.05$, so each rung represents a 5% productivity improvement. About half of laggard innovations are catch-up innovations ($\phi = 0.51$), but laggard catch-up is tempered by limited pledgeability ($\alpha = 3.81\%$). The values of σ_A and ρ jointly match the level of the financial cost of capital and the discount-rate gap. Overall, the fit between model-implied moments and the data is good. As shown in Figure 1, the model also fits key features of the full, nontargeted distributions of innovation output, markups, and discount-rate gaps.[25] Internet Appendix Table IA.1 reports the percent change in each targeted moment from a 1% increase in each indicated parameter or transformed parameter. The patterns are consistent with the identification discussion above.

Figure 2 illustrates the identification logic for two central innovation parameters, B and ϕ. A higher B lowers R&D costs in $G(x) = (x/B)^{1/\gamma}$, raising innovation incentives

[25]As described in Internet Appendix E.1, the model can better match the left tail of the wedge distribution if we focus on model-simulated panel data following a rise in κ due to higher idiosyncratic risk.

Figure 1: **Distributions of innovation output, markups, and discount-rate gaps**



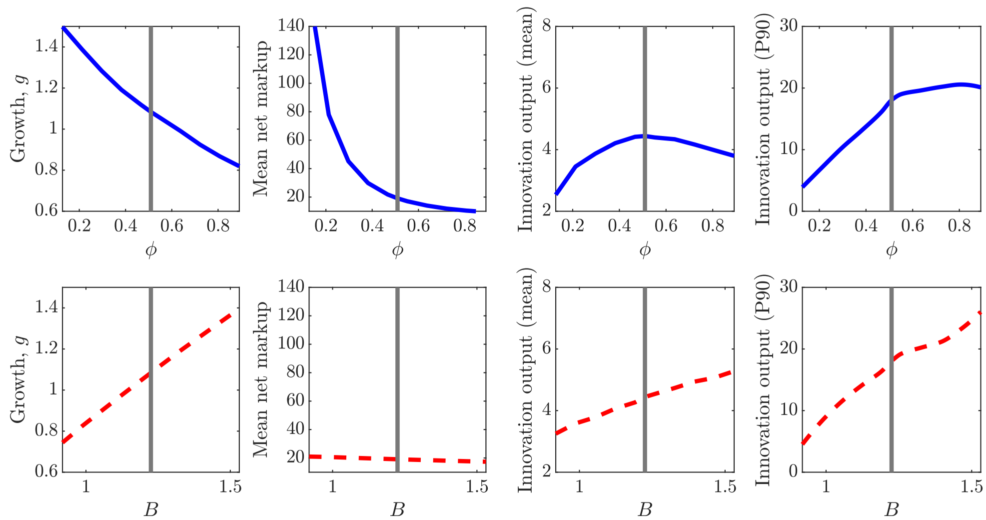
Data on innovation output, markups, and discount-rate gaps are from Kogan et al. (2017), Hall (2018), and Gormsen and Huber (2025), respectively.

for leaders, laggards, and tied firms. It thus raises growth and innovation output with little effect on the mean net markup. Higher ϕ changes the creative-destruction threat faced by leaders; it lowers markups, raises the right tail of innovation output, and reduces growth by diminishing leaders' incentive to pull further ahead.

4. Innovation, valuations, competition, and discount rates

A rise in uninsured idiosyncratic risk widens the gap between firms' discount rates and the financial cost of capital and generates lower growth, higher markups, and higher valuations—consistent with the trends motivating our paper. As we vary idiosyncratic risk, the financial cost of capital, hurdle rates, discount-rate gaps, productivity growth, and technology gaps all adjust endogenously. Figure 3 shows the asymmetric response of the key discount rates. As risk rises, hurdle rates rise moderately while the financial cost of capital falls more sharply. The cost of capital falls because of precautionary saving and the endogenous decline in growth. The hurdle rate rises because entrepreneurs require

Figure 2: **Relation of growth, markups, and innovation output with two innovation parameters**

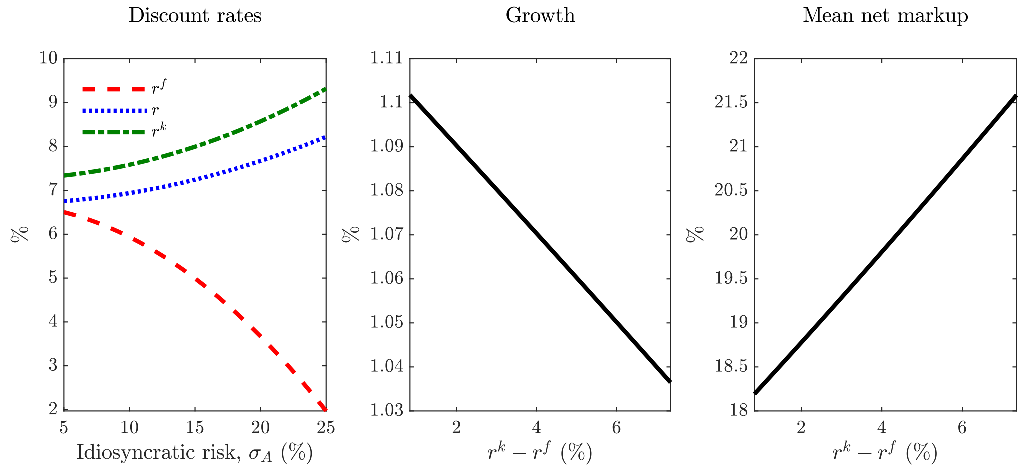


The figure shows how selected moments vary with laggard catch-up parameter ϕ and the R&D cost scalar B, holding other parameters fixed. The vertical line shows the calibrated parameter value. All moments are shown in percent.

greater compensation for undiversifiable risk and because greater risk endogenously tightens financing constraints by reducing the value of pledgeable claims to firm profits. This endogenous tightening is quantitatively important for the rise in the aggregate hurdle rate. Using Propositions 1 and 2, with a rise in σ_A from 5% to 25%, for example, about 74% of the increase in the hurdle rate reflects the greater compensation entrepreneurs require for bearing undiversifiable risk, while the remainder reflects the endogenous tightening of financing constraints. Even with the endogenous tightening, the cost of capital remains more sensitive to risk than the hurdle rate.

Greater exposure to undiversifiable idiosyncratic risk also reduces innovation and widens technology gaps, weakening market competition. Figure 3 shows that growth falls and the mean net markup rises as the discount-rate gap widens. Figure 4 shows the mechanism. Higher risk lowers innovation by both leaders and laggards. Because

Figure 3: **Discounts, productivity, and markups, varying idiosyncratic risk σ_A**



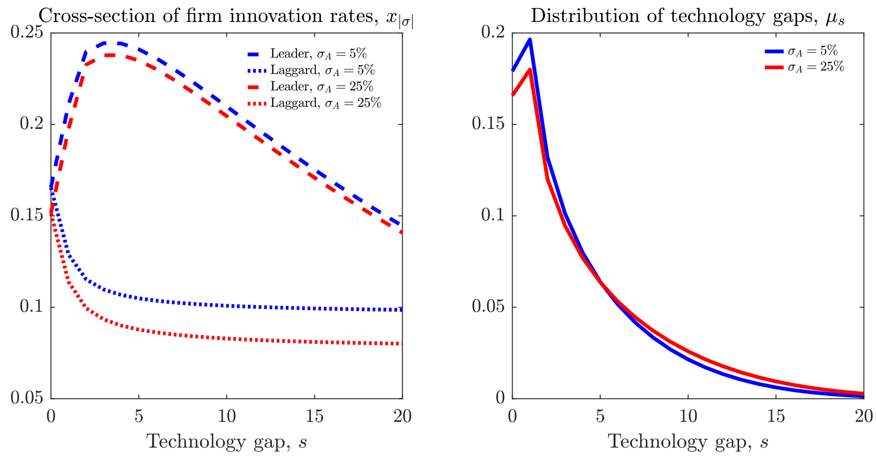
The left panel shows the relation of discount rates with idiosyncratic risk. The middle and right panels trace the relation of productivity growth and the mean net markup with the endogenous discount-rate gap, as we vary idiosyncratic risk σ_A.

innovating laggards obtain a mix of incremental and catch-up innovations, low laggard innovation induces a shift toward larger technology gaps and market leadership becomes more durable. Growth falls for two reasons: firms invest less, conditional on their current position, and the economy's composition shifts toward industries with wide technology gaps and low innovation.

Figure 5 shows the behavior of valuation ratios. Higher idiosyncratic risk raises the aggregate P/E ratio (Eq. (27)) and earnings-to-sales, so price-to-sales rises as well. Firm values increase despite weaker growth and despite firms forgoing innovations that, valued at the financial cost of capital, have positive present value. Higher hurdle rates reduce a firm's own R&D but also reduce rival innovation and therefore make market leadership more durable. Correspondingly, superstar firms (top 5%) see larger increases in firm values.

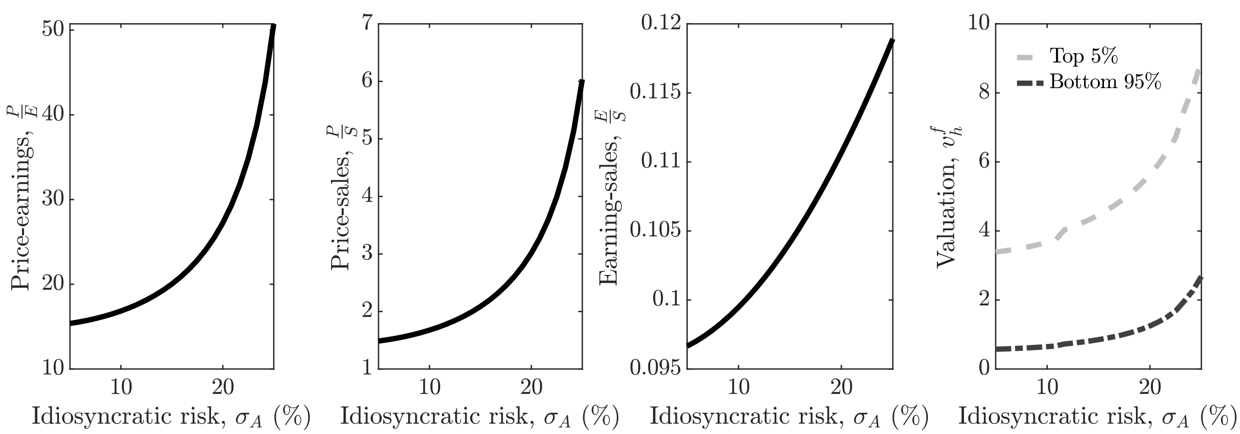
These patterns line up with the main macro-financial trends of recent decades. Mov-

Figure 4: **Firm innovation strategy and the distribution of technology gaps**



The left panel plots balanced-growth-path innovation rates $x(\sigma)$ by technology position σ for two values of idiosyncratic risk σ_A. The right panel plots the corresponding stationary distributions of technology gaps $\mu(s)$.

Figure 5: **Firm valuations and idiosyncratic risk**



Superstars are firms in the top 5% of the firm-value distribution; other firms are the remaining 95%.

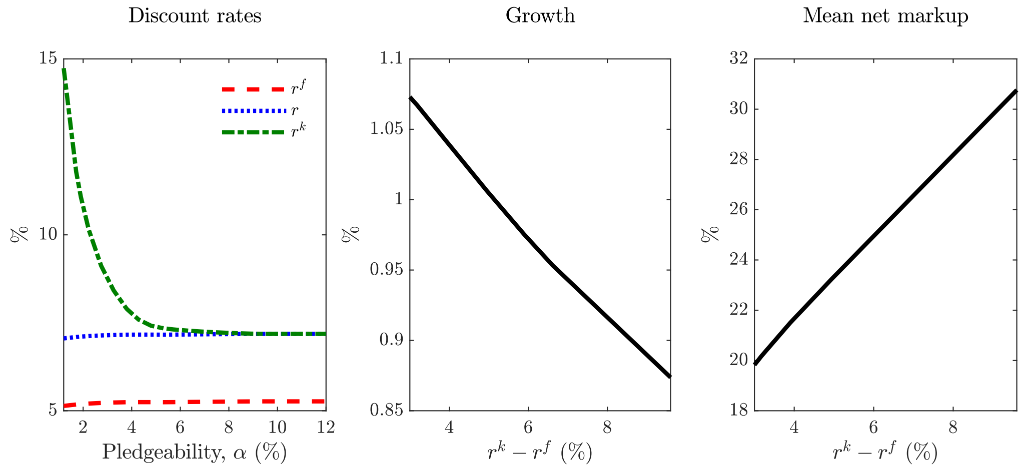
ing from $\kappa = 1\%$ to $\kappa = 6\%$—roughly the increase estimated by Gormsen and Huber (2025) for 1990 to 2021—the model implies a lower financial cost of capital, a moderate increase in hurdle rates, slower productivity growth (Fernald et al. 2017), higher markups, concentration, and profit share (Hall 2018; Autor et al. 2020; De Loecker et al. 2020; Barkai 2020), higher valuation ratios (Kuvshinov and Zimmermann 2022; Cho et al. 2026), especially for superstars (Corhay et al. 2025), and wider leader-laggard productivity gaps and weaker business dynamism more broadly (Akcigit and Ates 2021). Internet Appendix Figure IA.4 shows that a rise in idiosyncratic risk implies lower firm growth dispersion, lower job reallocation, and a lower labor share, consistent with the data.

The coexistence of weaker growth with a lower cost of capital is the opposite of the standard discounting channel in Schumpeterian models, in which a lower cost of capital raises the value of innovation and therefore increases growth. The mechanism is also distinct from Liu et al. (2022), in which a rise in households' savings demand reduces the growth rate, with the hurdle rate falling in tandem with the financial cost of capital.

A fall in pledgeability α also widens the discount-rate gap, lowers growth, and raises markups (Figure 6), but it does so through a different channel. Lower pledgeability raises the hurdle rate by tightening financing constraints, while the financial cost of capital changes little. Lower pledgeability affects the cost of capital only through lower growth in Eq. (23); unlike idiosyncratic risk, it does not generate the precautionary-savings motive that pushes the cost of capital down.

Entry. Declines in firm entry and the role of young firms are central dimensions of weakening business dynamism (Akcigit and Ates 2021). We therefore extend the model to include potential entrants, which hire $G_E(x_E) = (x_E/B_E)^{1/\gamma}$ R&D workers to enter at rate x_E, where B_E is entrants' R&D cost scalar. We re-estimate the model, augmenting the targeted moments with the entry rate, young-firm employment shares, and the net-entry component of the Foster et al. (2001) decomposition. The entry model fits these moments well while preserving the fit to growth, markups, discount rates, and key innovation moments. As in the baseline model, a rise in idiosyncratic risk σ_A endogenously raises

Figure 6: **Discounts, productivity, and markups, varying pledgeability α**



The left panel shows the relation of discount rates with the pledgeability parameter, α. The middle and right panels trace the relation of productivity growth and the mean net markup with the endogenous discount-rate gap, as we vary α.

κ, lowers the financial cost of capital, slows productivity growth, and raises valuations, especially for superstars; with entry, it also lowers entry and the contribution of net entry to productivity growth. These results link the endogenous discount-rate gap to a key dimension of declining dynamism. Internet Appendix F presents the entry analysis in detail.

Transition dynamics after higher idiosyncratic risk. Internet Appendix Figure IA.8 shows the transition dynamics following a one-time, unexpected, and permanent rise in idiosyncratic risk from $\sigma_A = 14\%$ to $\sigma_A = 21\%$. On impact, the financial cost of capital r^f falls, while the hurdle rate r^k rises only moderately. The discount-rate gap widens immediately, reflecting a higher idiosyncratic-risk premium and endogenously tighter borrowing constraints. Despite forgone positive-NPV projects, firm valuations v^f increase on impact: asset prices are forward-looking, and market values reflect the lower financial cost of capital, higher expected future markups, and weakened creative destruction. Productivity growth falls on impact because firms internalize the higher hurdle rate. During the transition, weak growth coexists with a low cost of capital and an elevated discount-rate gap—consistent with the across-BGP comparisons above.

To obtain these transition dynamics, we use an iterative shooting algorithm in which we guess the paths for the labor share and entrepreneurs' aggregate consumption growth; shoot backward to obtain scaled value functions and innovation rates; shoot forward to obtain the distribution of technology gaps; and update the guesses using labor-market clearing and the implied growth of entrepreneurs' aggregate consumption. Internet Appendix G describes the procedure in detail.

Sources of higher uninsured idiosyncratic risk. Greater exposure to undiversifiable risk can arise from increased sensitivity of decision-makers' payoffs to idiosyncratic outcomes (higher Λ), even if raw idiosyncratic risk is unchanged. In recent decades, high-powered, equity-based managerial incentives have become prevalent among public firms: the equity share of CEO compensation has risen from 5% in the 1990s to nearly 60% in 2025, while the salary share has declined from 40% to below 10%.[26] A shift in innovative activity to private firms and venture-capital-backed public firms—whose combined share of patents, importance-weighted by citations, increased from about 50% to 80% in recent decades—is also consistent with greater exposure of innovation decision-makers to uninsured idiosyncratic risk, as discussed in Internet Appendix B.

[26]Internet Appendix B connects these changes in the structure of CEO compensation to log increases in σ_A of 40% to 250%, through an approximate mapping to the model. Beyond the changing structure of CEO compensation, there has also been a well-documented increase in the level of CEO compensation.

5. Firm-level dynamics: Discount rates, R&D, and firm size

We next assess the model's cross-sectional predictions: the relation between discount-rate gaps and R&D, the evolution of gaps across firms with different market power, the firm characteristics associated with higher gaps, and firm dynamics related to creative destruction. These nontargeted patterns increase confidence that the estimated model captures the innovation and firm-heterogeneity channels through which discount-rate gaps affect aggregate growth and market power.

5.1 R&D and firm-level discount-rate gaps

A central mechanism in our model is that higher internal discount rates reduce R&D.[27] We therefore study whether hurdle rates and discount-rate gaps predict R&D using model-simulated panel data as well as Compustat data merged with firm-level hurdle-rate and gap estimates from Gormsen and Huber (2025).[28] The model panels are drawn from the balanced growth path of our benchmark calibration and from the transition following either an aggregate rise in uninsured idiosyncratic risk or a decline in the pledgeability of innovation profits. Table 2 reports regressions of annual log R&D-to-sales, measured one year ahead, on either the discount-rate gap or the hurdle rate. Higher gaps and hurdle rates predict lower future firm-level R&D intensity in the data, as in the model.

5.2 Secular increases in wedges and market power

Gormsen and Huber (2025) show that discount rates of firms with high market power have increased since 2002, relative to the discount rates of firms with low market power.

[27] Section 7 shows how well-documented changes in the innovation environment can offset this force in the aggregate, so that aggregate R&D can rise even when higher wedges reduce firm-level R&D and productivity growth.

[28] Internet Appendix A describes the construction of the discount rate series.

Table 2: **R&D and discount rates**

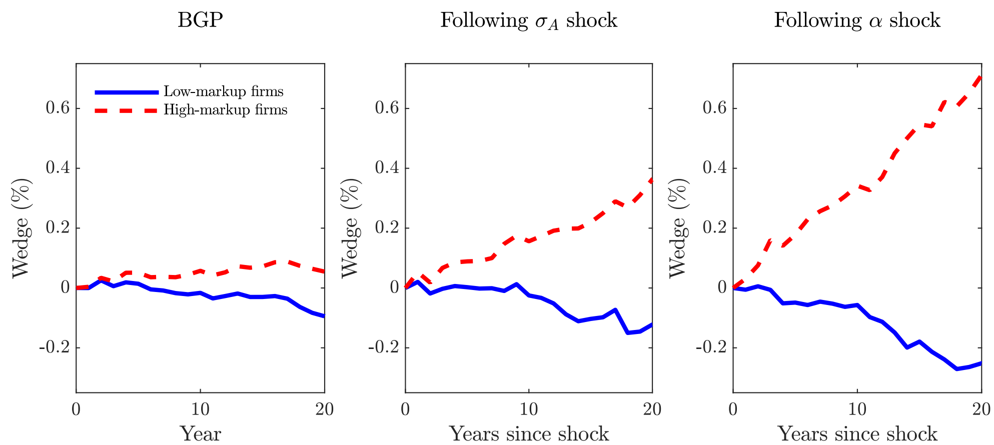
	Model			Data	
	Along BGP	Following σ_A shock	Following α shock	Gap	Hurdle
R&D-to-sales$_{t+1}$	-0.042	-0.032	-0.183	-0.033	-0.026
	(0.002)	(0.003)	(0.005)	(0.009)	(0.013)
Fixed effects	Firm + Year	Firm + Year	Firm + Year	Firm + SIC $\times$ Year	Firm + SIC $\times$ Year

This table reports firm-level panel regressions of annual log R&D-to-sales, measured one year ahead, on firms' discount-rate gap or hurdle rate in year t. Model columns use simulated panels along the balanced growth path and after the indicated aggregate shock (a 50% increase in risk σ_A or a 50% decrease in pledgeability α). In the model columns, the regressor is the discount-rate gap; because the financial cost of capital is common across firms at a given date, the hurdle rate has the same identifying variation after absorbing year fixed effects. In the data columns, the regressor is the discount-rate gap in the first column and the hurdle rate in the second. R&D-to-sales is Compustat XRD divided by SALE in the data. Discount-rate measures are from Gormsen and Huber (2025), as described in Internet Appendix A. Discount-rate variables are standardized to unit variance. Each model column has 44,000 observations; each data column has 10,119 observations. Standard errors, in parentheses, are clustered by firm.

Figure 7 shows the same cross-sectional pattern in the model, using simulations along the BGP and following the aggregate shocks. We split the sample based on firms' markups in year 0 of the simulation. In all three cases, discount-rate gaps rise for high-markup firms relative to low-markup firms. For each panel, we subtract the year 0 value from both series so that they start at zero, in order to isolate the cross-firm variation rather than the aggregate change in the wedge due to any shock (shocks are one-time and permanent, occurring at year 0).[29] Gormsen and Huber (2025) also show that firms facing tighter financial constraints according to the index of Hadlock and Pierce (2010) maintain higher discount rates and wedges; Internet Appendix Table IA.5 shows that this result obtains in model-simulated data as well.

[29]Gormsen and Huber (2025) also test these dynamics formally using panel regressions of firm-level discount rates and wedges on firm fixed effects and the interaction of market power and a time trend. We report the results of this regression in Internet Appendix Table IA.4. We obtain a positive coefficient estimate for this interaction term, as in the data. Also consistent with their paper, our Internet Appendix Table IA.5 shows, in model-simulated panels, that firms with more volatile idiosyncratic returns have higher gaps.

Figure 7: **Market power and the secular evolution of discount-rate gaps**



This figure plots the average discount-rate gap for firms sorted by their initial markup. Following Gormsen and Huber (2025), firms are assigned to high- and low-markup groups using the markup in year 0 of the simulation. We subtract the year 0 value from both series so that they start at zero.

5.3 Firm dynamics

Cross-sectional patterns in firm growth, transition rates across firm-size and wedge categories, and reallocation provide further ways to gain confidence in the model's approximation of the creative-destruction process, which affects the transmission of discount rates. We study four broad patterns related to firm dynamics and creative destruction. First, as Figure 8 (top row) shows, smaller firms have higher and more volatile growth rates in both the model and the data, consistent with well-documented patterns.[30]

Second, we examine transition rates across firm-size and wedge categories. As shown in Figure 8, bottom-left panel, market leadership is highly persistent in both the model and the data. Firms in the top half rarely fall to the bottom half the following year; firms in the top quintile usually remain there; and firms in the bottom 60% almost never reach

[30] Regarding the variance of growth rates, see, among others, Klette and Kortum (2004).

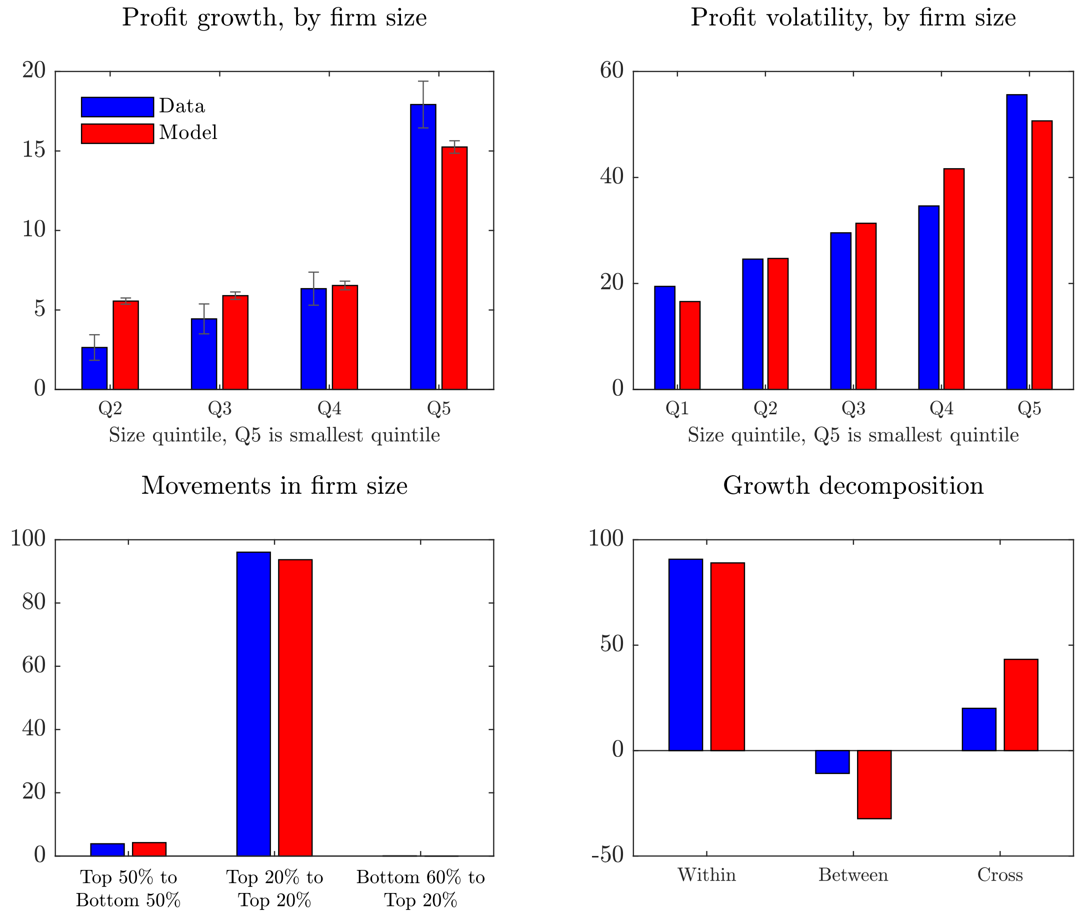
the top quintile. In the model and the data, wedges are also persistent, though less than firm size, as shown in Internet Appendix Figure IA.10.

Third, we study the Foster et al. (2001) decomposition of productivity growth among continuing firms. The within component captures productivity improvements holding market shares fixed, while the between and cross components together capture reallocation toward more productive firms. The cross component reflects the covariance between changes in revenue shares and productivity growth. Figure 8, bottom-right panel, shows that the model matches the decomposition in the data well (using values from Foster et al. 2008).

Fourth, we assess whether the model is consistent with Kogan et al.'s (2017) evidence that a firm's own innovation is associated with higher future profits and revenue-based productivity (TFPR), while innovation by competitors is associated with lower future profits and TFPR. Following their approach, using model-simulated data aggregated to an annual frequency, we estimate regressions of changes in profits and TFPR on a firm's own innovation output and its competitor's innovation output. Internet Appendix Table IA.6 reports the regression specification and coefficient estimates. The estimated model broadly matches the patterns in the data: own innovation output is associated with higher future profits and revenue productivity, while competitor innovation output is associated with lower profits and revenue productivity. The coefficient estimates in the simulated data are highly statistically significant, smaller than Kogan et al.'s (2017) estimates for TFPR, and larger than their estimates for firm profits.

Overall, these firm-level patterns support the margins through which our aggregate mechanism operates: discount-rate gaps are systematically related to firms' innovation

Figure 8: **Firm dynamics and creative destruction**



This figure compares model and data moments related to firm dynamics and creative destruction. The top-left panel plots coefficients from regressions of profit growth on firm-size quintile indicators, relative to firms in the top profit quintile. The top-right panel plots the standard deviation of profit growth by profit quintile. The bottom-left panel plots transition rates across firm-size categories. The bottom-right panel plots the Foster et al. (2001) decomposition of productivity growth for continuing firms. Firm size is measured by profits; Q1 is the largest-profit quintile and Q5 is the smallest-profit quintile. All moments are shown in percent.

decisions, and the model captures the creative-destruction dynamics that affect how discount-rate gaps transmit to productivity growth and business dynamism.[31]

[31] We also consider an additional source of firm heterogeneity in Internet Appendix H.3, which presents an extension in which firms have ex-ante heterogeneity with respect to the pledgeability of innovation profits.

Table 3: **Qualitative results for alternative channels**

	Prod. growth	Mean markup	Cost of capital	Discount-rate gap	P/E ratio	P/S ratio	Entry rate
Observed trend	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$
Higher pricing power	$\uparrow$	$\uparrow$	$\leftrightarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\leftrightarrow$
Barriers to entry	$\leftrightarrow$	$\uparrow$	$\leftrightarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$
Higher equity premium	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\leftrightarrow$

For three alternative mechanisms, this table compares the response of key variables with the observed trends. Upward arrows indicate increases, downward arrows indicate declines, and sideways arrows ($\leftrightarrow$) indicate minimal changes, defined as absolute log changes below 1% relative to the baseline balanced growth path. Entry-rate responses are computed in the model with entry (Internet Appendix F).

6. Alternative channels

We next use the model to distinguish our mechanism from three alternative channels—higher pricing power, rising barriers to entry, and a higher equity premium. None of these channels operates through the frictions that generate the discount-rate gap; in particular, absent idiosyncratic risk and limited pledgeability, the gap is zero regardless of pricing power or entry costs. Table 3 summarizes the qualitative responses: each alternative can match some moments, but none reproduces the joint behavior of productivity growth, the financial cost of capital, the discount-rate gap, markups, valuations, and entry.

Higher pricing power. The literature has linked an exogenous rise in market power to trends such as depressed investment in physical capital relative to levels implied by Tobin's Q (Gutiérrez and Philippon 2017; Eggertsson et al. 2021). In our setting, market power creates a wedge in firms' static production-labor decision: with Bertrand competition and imperfect substitution, each firm's gross markup $\psi_{z,j,t} \equiv p_{z,j,t}/(w_t/q_{z,j,t})$ strictly exceeds one, so the value of labor's marginal product exceeds the wage and production-labor demand is depressed (Internet Appendix C.1). This static product-market wedge is distinct from the intertemporal discount-rate gap that depresses R&D.

To generate an exogenous rise in markups, we follow Cavenaile et al. (2025) and increase the CES parameter ζ. A rise in ζ that raises the mean net markup by 5 percentage points raises productivity growth, as the prospect of higher profits spurs innovation; the financial cost of capital and the entry rate change little and, if anything, rise slightly (Table 3).[32] These responses miss the observed declines in growth, the cost of capital, and entry. With limited pledgeability, higher pricing power does raise the discount-rate gap, but only indirectly, through the components of the financing wedge in Eq. (22), namely, the scaled, weighted Lagrangian wedges. Overall, a rise in pricing power may matter for some outcomes, but, through the lens of our model, it is unlikely to be the dominant driver of low productivity growth, a falling cost of capital, a rising discount-rate gap, and declining entry.

Barriers to entry. The literature has also linked weakening business dynamism and declining market competition to rising barriers to firm entry (Akcigit and Ates 2023). We capture this channel through a decline in entrants' R&D productivity B^E in the calibrated model with entry (Internet Appendix F), lowering B^E enough to reduce the entry rate by 5 percentage points, in line with the fall in entry since the 1980s. As Table 3 shows, the entry barrier reduces entry and raises markups, but productivity growth and the financial cost of capital change little. Moreover, the discount-rate gap falls, contrary to the data: lower entry raises incumbents' profits and values, which loosens their financing constraints and reduces κ. Thus, higher entry barriers can help explain reduced entry

[32] A change in ζ has no direct effect on the financial cost of capital: in Eq. (23), ζ affects r^f only through equilibrium growth g. In contrast, higher idiosyncratic risk directly lowers the financial cost of capital by raising precautionary saving.

and dynamism, but not the broader joint trends motivating the paper.

Higher equity premium. Farhi and Gourio (2018) point to a higher equity premium as a key driver of weak investment in physical capital, falling risk-free rates, and a moderately rising return on private capital. A higher equity premium is a wedge between firms' financial cost of capital and the government-bond yield, rather than a wedge between firms' internal hurdle rates and their financial cost of capital (what we call the discount-rate gap). Because diversified claims on firms bear no aggregate risk in our model, we extend the model to capture this premium as a convenience yield: government debt provides safety or liquidity services that entrepreneurs value at a flow rate ϱ (Internet Appendix I). We pair the convenience yield with a rise of $\frac{1}{2}\varrho$ in entrepreneurs' time-preference rate, so that, for a given growth rate, the financial cost of capital rises by $\frac{1}{2}\varrho$ and the government-bond yield falls by $\frac{1}{2}\varrho$.[33] Entrepreneurs' required return rises by the same amount, so the convenience wedge introduces no direct gap between hurdle rates and the cost of capital.

We study a rise in ϱ of 4 percentage points. This channel lowers productivity growth and raises markups, but it raises the financial cost of capital and lowers the P/E and price-to-sales ratios; the entry rate is little changed. The discount-rate gap rises, but only indirectly, through the financing wedge. Thus, through the lens of our framework, a higher equity premium moves valuations and the financial cost of capital in the wrong direction.

[33]The government-bond yield becomes $r^{\text{gov}} = \bar{r} - \frac{1}{2}\varrho$ and the financial cost of capital becomes $r^f = \bar{r} + \frac{1}{2}\varrho$, where $\bar{r} \equiv \rho + \frac{g}{\epsilon} - \frac{1}{2}\left(1 + \frac{1}{\epsilon}\right)\nu\sigma_A^2$ is the cost-of-capital expression in Eq. (23).

7. Ideas harder to find

The preceding sections show that greater exposure to undiversifiable idiosyncratic risk widens the aggregate discount-rate gap, lowers the financial cost of capital, and weakens productivity growth, market competition, and business dynamism. The model's central firm-level channel—higher discount-rate gaps predict lower R&D—appears in both the data and model-simulated panels, along with other relevant nontargeted patterns. The aggregate R&D time series raises an additional issue: the R&D to GDP ratio has risen while productivity growth has fallen, a pattern often described as ideas becoming harder to find (Bloom et al. 2020). In our framework, changes in discount-rate gaps alone move aggregate research effort and productivity growth in the same direction—higher gaps discourage R&D and reduce growth—so the observed divergence points to concurrent changes in the innovation environment. The Schumpeterian literature has identified such changes, including weaker laggard catch-up (Akcigit and Ates 2023; Olmstead-Rumsey 2025) and a greater role for scale in R&D due to specialization and team science (Kim et al. 2026).

We incorporate these forces by allowing four parameters to change. The first is ϕ, the probability that a successful laggard innovation closes the gap with the leader; a lower ϕ makes catch-up less likely and tends to widen technology gaps. The second is the R&D cost-curvature parameter γ; a higher γ makes R&D costs less convex and gives scale a larger role in producing innovations. The third is the R&D cost scalar B, which lets the calibration change cost curvature without mechanically imposing a particular change in the level of R&D costs. The fourth is idiosyncratic investment risk, σ_A, which

Table 4: **Calibrated changes in parameters and targeted moments**

Change in parameters		Change in moments		
Parameter	Change	Moment	Model	Data
ϕ	-14.50%	Discount-rate gap κ	82.90%	95.94%
γ	35.64%	Productivity growth g	-23.28%	-23.36%
B	19.54%	R&D to GDP	24.03%	28.80%
σ_A	44.64%	Mean net markup	9.67%	16.78%

All entries are percent changes. Model entries compare year 20 of the transition with the initial balanced growth path; data entries compare the 1990s and 2010s decade averages of each moment, as described in Internet Appendix D.

the preceding sections show can widen the discount-rate gap while lowering the financial cost of capital.

We discipline the change in $(\phi, \gamma, B, \sigma_A)$ using the two-decade changes in four aggregate moments: the discount-rate gap, productivity growth, R&D to GDP, and the mean net markup. Each data target is the percent change in the moment's decade average, comparing the 1990s and 2010s; each model target is the corresponding percent change between year 20 and the initial BGP.[34] For the discount-rate gap, we use Gormsen and Huber (2025), augmented with their mapping of the Poterba and Summers (1995) data; for productivity growth, Fernald et al. (2017); for the mean net markup, Autor et al. (2020); and for R&D to GDP, National Income and Product Accounts data from the Bureau of Economic Analysis.

Matching the observed changes requires weaker laggard catch-up, a more scale-intensive R&D technology, and higher idiosyncratic risk, as shown in Table 4. The fall in

[34]The Gormsen and Huber (2025) data begin in 2002. We extend the series to 1990 using their mapping of Poterba and Summers (1995), which places the 1990 wedge 2.3 percentage points below the 2003 value. Using the 2003 value of 3.58% (consistent with their Figure 6), this approach implies a κ value of 1.28% in 1990. Linear interpolation to 2003 then yields a 1990s average of 2.08%. The 2010s average is 4.07%, implying a 96% increase, which is our target. Because κ rose within each decade, the 1990-to-2021 endpoint change is larger, from 1.28% in 1990 to 5.58% in 2021. This is summarized as 1% to 6% in Section 4. Our benchmark calibration matches the cross-sectional mean of κ (2.64%), which serves as the initial condition for the transition. At date 0 the four parameters jump permanently to their new values.

ϕ captures weaker catch-up by laggards. The rise in γ increases the role of scale in R&D. The rise in B should not be interpreted as ideas becoming easier to find; it offsets the mechanical level effect of changing γ, allowing the exercise to match the observed rise in R&D to GDP while changing the scale dependence of R&D. The rise in σ_A generates the higher discount-rate gap and contributes to lower productivity growth and higher markups. Together, these changes reproduce the central ideas-hard-to-find pattern: productivity growth falls even as R&D to GDP rises. Quantitatively, the model nearly matches the observed decline in productivity growth, captures most of the increases in the discount-rate gap and R&D to GDP, and captures a substantial share of the increase in the mean net markup. The transition paths underlying these year-20 moments are shown in Internet Appendix Figure IA.9.

The analysis also clarifies the role of the paper's financial frictions. Changes in ϕ, γ, and B help explain why aggregate research effort can rise while growth falls. However, such changes, by themselves, do not explain the discount-rate facts. If idiosyncratic risk and the pledgeability constraint are removed, the discount-rate gap is zero and constant, and changes in catch-up or the R&D technology affect the cost of capital only through the equilibrium growth-rate term in Eq. (23). Thus, the discount-rate mechanism is financial, while changes in the innovation environment account for the decline in aggregate research productivity. Overall, this analysis reconciles the firm-level evidence emphasized above—higher wedges are associated with lower R&D-to-sales—with the aggregate fact that R&D to GDP has risen while productivity growth has fallen.

As a final application, we show that the transition dynamics in this section are consistent with the aggregate and within-firm relations among firm valuations, markups, the cost of capital, and intangible capital documented by the present-value decomposition of Cho et al. (2026). Asset prices in the model are forward-looking, so changes in financial frictions (higher uninsured idiosyncratic risk) and in the innovation environment (ideas harder to find) affect firm values through both expected cash flows and the financial cost of capital. Cho et al. (2026) attribute the rise in market value-to-output (in our model, price-to-sales) to higher expected markups and a lower financial cost of capital, partly offset by higher expected intangible investment. The calibrated transition produces the same pattern: with the change in $(\phi, \gamma, B, \sigma_A)$, aggregate price-to-sales rises substantially, reflecting higher expected markups and a lower financial cost of capital, partly offset by higher expected R&D. Internet Appendix Table IA.7 shows that the same pattern obtains within firms, in the model and the data. We apply the Cho et al. (2026) decomposition to firm-level data simulated along the transition and find that within-firm valuation gains are associated with higher markups and a lower cost of capital, with higher R&D limiting these gains, as in their within-firm estimates. Thus, the cross-sectional patterns they document using a reduced-form log-linear present-value identity emerge here from a creative-destruction model with an endogenous discount-rate gap.

8. Conclusion

Firms' hurdle rates exceed the financial cost of capital, and this gap has widened as the cost of capital has fallen. We develop a Schumpeterian model in which this wedge varies across firms and arises endogenously from undiversifiable innovation risk and imperfect pledgeability of innovation profits. The framework separates the discount rate governing firms' R&D decisions from the discount rate that financial markets use to value cash flows, and studies how that gap reshapes creative destruction and strategic competition among market leaders, laggards, and entrants.

A main result is that greater exposure to undiversifiable idiosyncratic risk generates a notable fall in the financial cost of capital, a moderate rise in firms' hurdle rates, slower growth, and weaker dynamism. The higher discount-rate gap reduces leader and laggard innovation, weakens creative destruction, and makes market leadership more durable. Profits and superstar valuations rise even though growth falls and firms forgo projects that would have positive net present value when discounted at the financial cost of capital.

The calibrated model matches aggregate and cross-sectional moments related to discount rates, R&D, innovation output, firm dynamics, markups, and growth. It also reproduces nontargeted patterns linking wedges to R&D and financial constraints; cross-firm variation in market power helps explain the evolution of wedges, as in the data. With higher idiosyncratic risk and well-documented changes in the innovation environment (weaker laggard catch-up and a more scale-intensive R&D technology), the model can also account for slower productivity growth despite rising R&D. More broadly, the paper offers a view of firm valuations and growth in which financial frictions and strategic creative destruction are intertwined. As a result, a low financial cost of capital can coexist with sticky and elevated firm hurdle rates, slow productivity growth, weak business dynamism, and high firm profits and valuations.

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Internet Appendix for

"The innovator's risk premium: Sticky hurdle rates, the cost of capital, and creative destruction"

A. Discount rates and the return on business capital

Our paper uses data on hurdle rates from Gormsen and Huber (2025), who measure firms' discount rates and cost of capital by manually recording these values from a large set of corporate conference calls. Their approach builds on and is complementary to Poterba and Summers (1995), who use a CEO survey of discount rates. A crucial issue in using conference calls is that some firms use discount rates that are adjusted upward to compensate for some overhead costs, such as administrative or headquarters costs, that are omitted from cash flow analyses. To address this issue, following Gormsen and Huber (2025), we focus on discount rates for firms that fully account for overhead in their cash flow analyses. To obtain our target for aggregate κ, we take the mean across firms of the difference between the discount rate and cost of capital. Throughout the paper, we use hurdle rates and cost of capital from Gormsen and Huber's (2025) updated and publicly available data set, which implies a mean κ of 2.64%.[35] These hurdle rates and costs of capital are predicted values, based on Lasso regressions using the manually recorded rates together with 153 firm-level characteristics (from Jensen et al. 2023). Using the mean values reported in Gormsen and Huber (2025) (an average nominal corporate discount rate of 11.5% for firms that include all overhead costs in their cash flow analyses, and an average perceived cost of capital of 8.4%), we obtain a very similar value for mean κ, 3.1%. These values of κ are somewhat below the implied κ from Poterba and Summers (1995), who report division-level discount rates for cash flow analyses that mostly do not account for overhead, per Gormsen and Huber (2025).

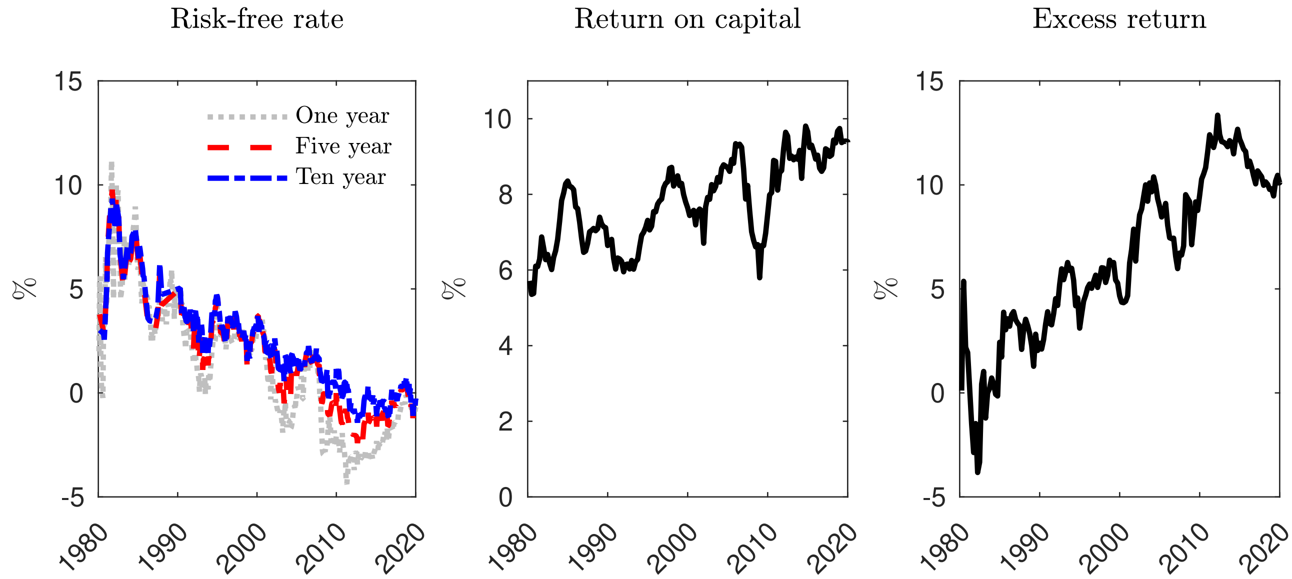
Several factors motivate our use of Gormsen and Huber's (2025) publicly available data set of predicted hurdle rates, costs of capital, and discount-rate gaps. They validate the predicted values using the Duke-CFO survey (Graham and Harvey 2001): regressing the perceived cost of capital from the Duke-CFO survey on their predicted values yields a slope close to 1, so cross-sectional variation in the predicted values is reflected approximately one-for-one in the survey.

Furthermore, because all firm characteristics in the model are a function of the discrete firm-level state variable σ (which can take only $2\bar{s}+1$ values), applying Gormsen and Huber's (2025) regression approach to model-simulated data would produce (under the conditions in the footnote) a linear model that is exactly right, driving the residual sum of squares to zero and pushing R^2 to 1, making predicted discount rates equal to actual discount rates in the model.[36] Thus, model-implied κ and model-predicted κ

[35] Obtained from https://costofcapital.org/, Version 2.0, updated January 11, 2024, with conference call data through September 2023.

[36] Let $\kappa_{f,t}$ denote the discount-rate gap of firm f at time t. Along a BGP, $\kappa_{f,t}$ and model-generated firm characteristics are deterministic functions of the firm's technology gap $\sigma_{f,t} \in S$. Because the gap distribution is stationary, the percentile ranks of these characteristics are fixed functions $p(\sigma)$. If the percentile-rank

Figure IA.1: **Real interest rate and return on capital**



This figure updates the evidence on the risk-free rate and the return on capital in Figure 1 of Caballero et al. (2017). The left panel shows measures of the risk-free rate. The 1-year risk-free rate is the one-year constant-maturity nominal Treasury yield less four-quarter inflation according to the lagged GDP deflator. The 5- and 10-year rates shown are the nominal constant-maturity Treasury yield for the indicated horizon, less expected inflation according to the Michigan survey. The middle panel shows the return on capital based on Gomme et al. (2011). The right panel shows the difference between the return on capital and the one-year risk-free rate.

effectively coincide in the model.

Trend in κ and realized return on business capital. Gormsen and Huber (2025) find that κ has increased by about 2.5 percentage points between 2002 and 2021. Connecting their estimates with Poterba and Summers (1995), assuming a constant overhead fraction, Gormsen and Huber (2025) find a rise in κ between 1990 and 2003 of 2.3 percentage points. Similarly, Caballero et al. (2017) and Farhi and Gourio (2018) provide empirical estimates (based on Gomme et al. 2011) showing a rising gap between the realized return on business capital and safe interest rates. Hall (2015) provides additional evidence of this trend. Figure IA.1 provides updated estimates. Following Caballero et al. (2017) and Farhi and Gourio (2018), we measure the return on capital as the after-tax return on business capital from Gomme et al. (2011), adjusted for intangibles.[37] The return on

vectors of the occupied states are affinely independent (which requires at least as many characteristics as technology positions σ with $\mu(|\sigma|) > 0$, minus one), then any state-dependent object, including $\kappa_{f,t}$, can be written exactly as an affine function of the characteristics. Thus, with many firms in each state and a penalty that vanishes with sample size, the Lasso regression attains this exact representation and its in-sample R^2 converges to one whenever κ varies with a firm's technology position.

[37]Following Caballero et al. (2017) and Farhi and Gourio (2018), we adjust for intangibles by adding to

capital has increased over time, amid a widening gap with falling short- and long-term interest rates.

B. Sources of uninsured idiosyncratic risk

B.1 Trends in the structure of CEO compensation

A rise in idiosyncratic risk borne by firm decision-makers is consistent with the significant shift toward equity-based and high-powered managerial incentives, a central corporate-governance trend in recent decades.[38] This section provides evidence on this shift and relates changes in the structure of CEO compensation to changes in the model's uninsured idiosyncratic-risk parameter, σ_A.

Figure IA.2 provides updated evidence from Edmans et al. (2017) on the changing structure of CEO compensation. Since 1990, stock-based compensation has increased sharply, while salary compensation has declined notably, as a share of CEO compensation. The solid black line shows our own calculations using Compustat ExecuComp data. Our results align closely with the values reported in Edmans et al. (2017), which are available only through the early 2010s. Our updated evidence shows that, since then, the changes in the structure of CEO compensation have become even more pronounced.

Next, we relate changes in the structure of CEO compensation to changes in the model's uninsured idiosyncratic-risk parameter, σ_A. As described in the model section, with constant parameters, $\sigma_A = \Lambda\tilde{\sigma}_A$, where Λ captures limits to risk sharing and $\tilde{\sigma}_A$ represents raw idiosyncratic uncertainty (see Section 1.4 and Internet Appendix C.3). Our model accommodates deterministic time-variation in parameters (see Internet Appendix G) and we can also study unexpected shocks to the parameter path. Thus, we can write $\sigma_{A,t} = \Lambda_t\tilde{\sigma}_{A,t}$. For CEOs, retained shares, unvested equity, and stock grants are sensitive to firm value (consistent with $\Lambda_t > 0$), while current salary and cash savings are relatively insulated from firm-level idiosyncratic returns. Moreover, the changing structure of CEO compensation is informative about changes in Λ_t.[39]

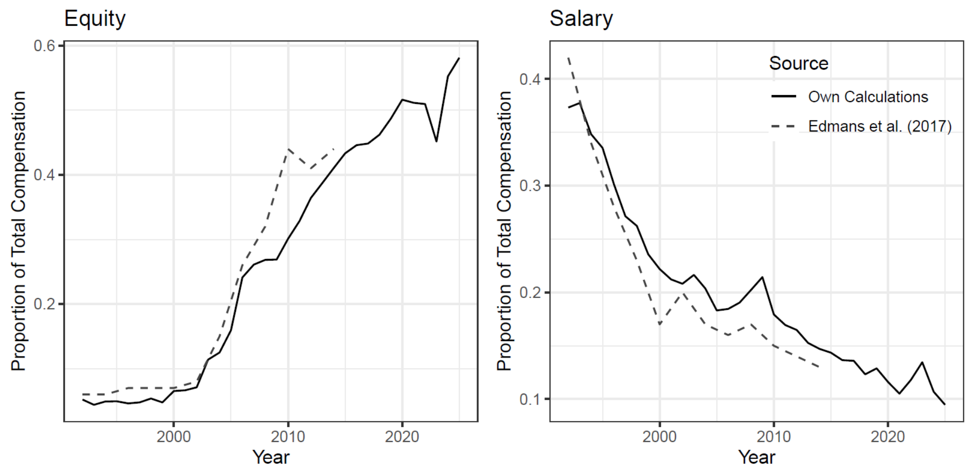
CEO wealth can be decomposed into components with different firm-value loadings. Cash savings and fixed cash compensation have loading close to zero. Retained or

the overall business capital stock the current cost net stock of fixed assets for intellectual property products, converted to real cost using the same deflator as in Gomme et al. (2011).

[38] Ma and Shleifer (2025) describe the growth of equity-based incentives as an important consequence of the modern corporate-governance revolution. Frydman (2018) describes a shift after the mid-1980s from salary compensation toward equity-based pay, while Hall and Liebman (1998) show that stock holdings are central to the sensitivity of CEO wealth to firm performance.

[39] Note that σ_A^2 enters the model multiplied by decision-makers' relative risk aversion. Risk aversion does not enter elsewhere because the model has no aggregate risk. Thus, for example, a rise in σ_A^2 from 0.01 to 0.04 with relative risk aversion equal to one has the same implications as a rise in σ_A^2 from 0.02 to 0.08 with relative risk aversion equal to one half. For this reason, this section focuses on log changes in σ_A. See also footnote 23.

Figure IA.2: **Changing structure of CEO compensation**



This figure shows equity-based and salary compensation as a share of CEO compensation, over time. The solid black line is calculated using the Compustat ExecuComp Annual Compensation data set, restricted to S&P 500 firms (SPCODE = SP) and CEO positions (CEOANN = CEO). Equity compensation is defined as the sum of RSTKGRNT and STOCK_AWARDS_FV. The former captures the grant-date value of restricted stock under the 1992 reporting format, while the latter captures the grant-date fair value of all stock awards under the revised FAS 123R accounting rule's reporting format change (late 2005 onward). Summing these variables is consistent with the staggered adoption of the new reporting requirements across firms; this does not result in double-counting because firms report under only one standard in any given year. Salary compensation uses the SALARY variable directly. Both components are divided by TDC1, total compensation, to create firm-level ratios. For each year, the mean equity-to-total and salary-to-total ratios are calculated across all S&P 500 CEOs in the sample. The dashed line shows values from Edmans et al. (2017).

unvested equity has loading close to one. Future compensation has positive loading to the extent that it responds to today's firm-value shock. Future equity awards have little loading if they are fixed-dollar grants set after the shock, but high loading if they are fixed-share awards, performance-contingent awards, or otherwise proportional to firm value. Future salary may also load on firm value through retention, promotion, outside-option, or firm-size effects, though we treat salary as insulated here.

Let e_t denote the equity share of compensation, and let s_t denote the salary share of compensation. We use the equity share e_t as a benchmark proxy for the performance-sensitive share of compensation-generated wealth. This approximation is more natural when past compensation accumulates into cash and retained equity in roughly the same exposed and insured proportions as current compensation, as when CEOs consume similarly out of past cash and past equity compensation. Then a higher equity share means that more accumulated compensation-generated wealth is held in a form that is sensitive to firm value. Similarly, future equity awards may load on current changes in firm value, as with awards of a fixed number of shares. Then a higher equity share also means that more of the present value of future compensation is exposed to firm-value risk.

Under this benchmark,

$$\Lambda_t \propto e_t,$$

where the constant of proportionality reflects the share of CEO wealth outside the firm-specific compensation stake, such as unrelated diversified wealth. If this outside-wealth share is stable, it affects the level of Λ_t but not its growth. Holding underlying idiosyncratic firm-value volatility ($\tilde{\sigma}_A$) fixed,

$$\log \sigma'_A - \log \sigma_A \approx \log e' - \log e, \tag{IA.1}$$

where primes denote values after the change in the compensation structure. This equity-share calculation is one way to proxy for the change in CEO exposure to firm-specific risk.

An alternative proxy uses the non-salary share, $1 - s_t$. This salary-focused proxy treats salary as the insured component of compensation and all non-salary compensation as exposed to firm performance:

$$\Lambda_t \propto 1 - s_t.$$

Under this alternative,

$$\log(\sigma'_A) - \log(\sigma_A) \approx \log(1 - s'_t) - \log(1 - s_t). \tag{IA.2}$$

The difference between the two proxies is the treatment of non-salary, non-equity compensation. Using e_t treats this component as less sensitive to firm performance than equity compensation. Using $1 - s_t$ treats it as having the same loading as equity compensation. In practice, this category is heterogeneous: it includes options, whose loading on firm value depends on moneyness and option characteristics, and also personal benefits and tax reimbursements, which are fairly insensitive to firm performance. The equity-share and non-salary-share mappings therefore provide two useful benchmarks.

Figure IA.2 shows a large change in the structure of CEO compensation. The equity share rises from about 5% in the 1990s to about 60% in the later sample. Using Eq. (IA.1),

$$\log(0.60) - \log(0.05) = 248.5\%.$$

The salary share falls from about 40% in the 1990s to about 10% toward the end of the sample. Using Eq. (IA.2),

$$\log(1 - 0.10) - \log(1 - 0.40) = \log(0.90) - \log(0.60) = 40.5\%.$$

Thus, the rise in the equity share and the fall in the salary share shown in Figure IA.2 are consistent with potentially large increases in σ_A: log increases upwards of 40% and as high as 250%. (For comparison, the transition exercise in Section 7 features an increase in σ_A of 44.6%, equivalent to a log increase of 36.9%.)

B.2 Innovation by private and VC-backed firms

A related trend that can raise innovation decision-makers' exposure to uninsured idiosyncratic risk is a shift in the composition of innovative activity toward private firms and the rising importance of venture capital and venture-capital-backed firms. Private firms are typically characterized by more concentrated cash-flow rights and control rights than public firms, and their founders, managers, and specialist investors often cannot fully diversify project-specific risk. This distinction is relevant because high-powered incentives can expose managers to idiosyncratic risk, and (among public firms) investment is more sensitive to firm-specific risk when managers own larger stakes (Panousi and Papanikolaou 2012). Well-known examples illustrate the trend: significant innovation has been driven by private firms in space, software, and artificial intelligence businesses, and at formerly VC-backed public firms such as Tesla, Meta, and Alphabet whose founders retained substantial decision rights and exposure to firm value after listing. More broadly, public listings have declined over recent decades (Doidge et al. 2017), while private capital has expanded and dual-class IPOs increasingly give founders and insiders voting rights in excess of their cash-flow rights (Aggarwal et al. 2022).

Here, we document a compositional shift in innovative activity and discuss it through the lens of the literature. We start from all corporate utility patents in PatentsView and weight each patent by citation-adjusted importance, defined as $1+$ forward citations normalized by the filing-year mean. We classify public-company patents using the KPSS/CRSP bridge (Kogan et al. 2017). Private firms are the converse (corporate assignees that are not public firms in the filing year). The private share rose from one-half (51%) of citation-adjusted corporate patent importance in 1980 to almost three-quarters (72%) by 2021.

This shift took place amid the emergence of the venture capital industry. Total dollars invested by venture capital across all rounds of financing for U.S. firms (adjusted to 2021 dollars using the Consumer Price Index) rose from \$7 billion in 1985 to \$360 billion in 2021.[40] In addition, VC influence often extends beyond the transition to public ownership. Iliev and Lowry (2020) show that approximately 15% of VC-backed firms raise additional capital from VCs during the five years after going public, consistent with specialist investors continuing to finance high-growth public firms. VC-backed public firms are also central to U.S. innovative activity: Gornall and Strebulaev (2021) document that VC-backed public companies account for 41% of total U.S. market capitalization and 62% of U.S. public-company R&D spending; among public companies founded within

[40] These data are from the 2013 and 2026 National Venture Capital Association's Yearbooks, updating and consistent with the calculation in Lerner and Nanda (2020).

the previous fifty years, they account for more than 92% of R&D spending and patent value (see also Lerner and Nanda 2020). These facts point to a shift in innovative activity toward firms shaped by concentrated specialist investors and, in many cases, founders and insiders with concentrated exposure and strong control rights.

Related evidence links these governance, funding, and incentive arrangements to innovation. Among VC-backed IPO firms, Tian and Wang (2014) show that backing by more failure-tolerant VCs is associated with greater innovation, especially when ventures face high failure risk. Among newly public firms, Baranchuk et al. (2014) find that innovative firms use more CEO incentive compensation and longer vesting periods, and that these features are associated with subsequent patenting. Incentives also matter below the executive level: Chang et al. (2015) show that non-executive employee stock options are positively associated with patent counts and patent citations.

Figure IA.3 documents the compositional shift in innovative activity. Beyond the classification into public and private firms described above, we further separate public firms into VC-backed and non-VC-backed firms using Jay Ritter's IPO data (Ritter 2015).[41] The figure plots the share of citation-adjusted corporate patent importance accounted for by private firms and VC-backed public firms. The private-or-VC-backed-public share rises from a bit above one-half of citation-adjusted corporate patent importance in 1980 to more than four-fifths by the early 2020s. Thus, patenting activity has shifted away from non-VC public firms and toward private firms and VC-backed firms, a composition consistent with greater exposure of innovation decision-makers to uninsured idiosyncratic risk.

C. Model details, proofs, and equilibrium

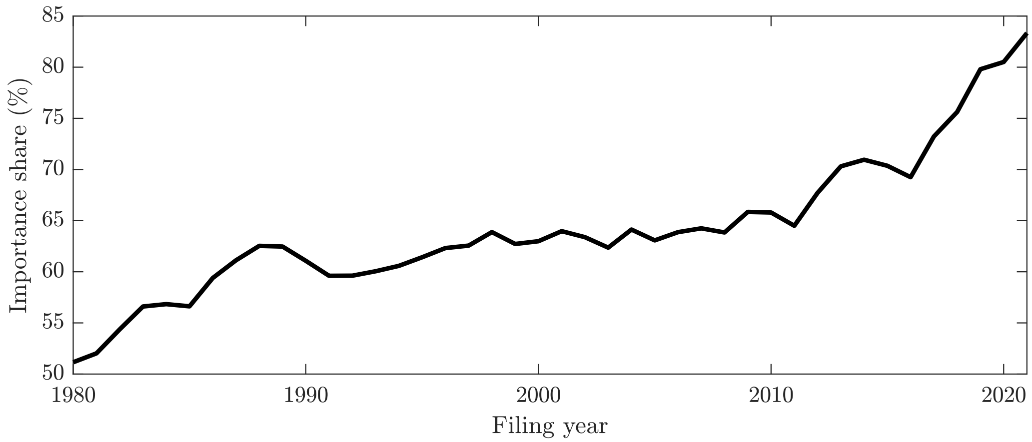
C.1 Intermediate goods market

This subsection derives the static Bertrand equilibrium within an industry: the relative-price equation and the profit expressions in Section 1, together with the markups and labor demands used throughout the paper. Lemma IA.1 shows that the relative price exists, is unique, and is decreasing in the technology gap, which implies the monotonicity of profits and markups in the technology position. Lemma IA.2 characterizes the perfect-substitutes limit $\zeta \to 1$ and verifies the limit-pricing claims in the main text.

Demand and revenue shares. Consider an industry whose firms have productivities $q_L \geq q_F$, where L denotes the leader and F the laggard ("follower"); in a tied industry, the labels are arbitrary. Firm $z \in \{L, F\}$ has marginal cost w/q_z and chooses its price p_z, taking its competitor's price as given. As stated in Section 1, cost minimization by

[41] We treat both standard VC-backed IPOs and growth-capital-backed IPOs as VC-backed, though results are very similar when restricting to pure VC-backed IPOs.

Figure IA.3: **Private and VC-backed share of patenting (weighting by patent importance using citations)**



This figure plots the share of citation-adjusted corporate patent importance accounted for by private firms and VC-backed public firms. Patent importance is defined as $(1+$ forward PatentsView citations) divided by the filing-year mean. Public companies are identified using the KPSS/CRSP bridge (Kogan et al. 2017). VC-backed public firms are identified using Jay Ritter's IPO data (Ritter 2015).

the final-good producer implies the within-industry demand condition $\frac{p_L}{p_F} = \left(\frac{y_L}{y_F}\right)^{\zeta-1}$, and Cobb–Douglas aggregation implies that revenue in each industry equals aggregate output Y. Let $m \equiv p_L/p_F$ denote the leader-to-laggard relative price, and let $\delta_z \equiv p_z y_z/Y$ denote firm z's share of industry revenue. The demand condition implies $\delta_L/\delta_F = m^{-\frac{\zeta}{1-\zeta}}$, which combined with $\delta_L + \delta_F = 1$ gives

$$\delta_L = \frac{1}{1+m^{\frac{\zeta}{1-\zeta}}}, \qquad \delta_F = \frac{m^{\frac{\zeta}{1-\zeta}}}{1+m^{\frac{\zeta}{1-\zeta}}}. \tag{IA.3}$$

Because $m \leq 1$ in equilibrium (Lemma IA.1), the leader has a weakly larger revenue share, strictly larger when $q_L > q_F$ (i.e., outside tied industries).

Prices and markups. Holding the competitor's price fixed, firm z's demand is $y_z = Y p_z^{-\frac{1}{1-\zeta}} / \left(p_L^{-\frac{\zeta}{1-\zeta}} + p_F^{-\frac{\zeta}{1-\zeta}}\right)$, so the price elasticity of demand facing firm z is

$$\eta_z \equiv -\frac{d \ln y_z}{d \ln p_z} = \frac{1-\zeta\delta_z}{1-\zeta} = (1-\delta_z)\,\frac{1}{1-\zeta} + \delta_z. \tag{IA.4}$$

Because $\delta_z < 1$, we have $\eta_z > 1$, so the Bertrand first-order condition is the Lerner rule $\frac{p_z - w/q_z}{p_z} = \frac{1}{\eta_z}$, and the gross markup $\psi_z \equiv \frac{p_z}{w/q_z}$ is

$$\psi_z = \frac{\eta_z}{\eta_z - 1} = \frac{1-\zeta\delta_z}{\zeta(1-\delta_z)}, \tag{IA.5}$$

which is increasing in δ_z. Substituting the shares in Eq. (IA.3),

$$\psi_L = \frac{1+(1-\zeta)m^{-\frac{\zeta}{1-\zeta}}}{\zeta}, \qquad \psi_F = \frac{1+(1-\zeta)m^{\frac{\zeta}{1-\zeta}}}{\zeta}.$$

Taking the ratio of the two pricing rules $p_z = \psi_z\, w/q_z$ yields the equilibrium condition for the relative price:

$$m = \frac{q_F}{q_L}\left(\frac{1+(1-\zeta)m^{-\frac{\zeta}{1-\zeta}}}{1+(1-\zeta)m^{\frac{\zeta}{1-\zeta}}}\right). \tag{IA.6}$$

With $q_L/q_F = \lambda^s$, Eq. (IA.6) is the equation that defines m_s in Section 1. Given m, the pricing rules then determine the price levels. In a tied industry, $m = 1$, each firm's revenue share is $\frac{1}{2}$, and $\psi_L = \psi_F = \frac{2-\zeta}{\zeta}$.

Profits and labor demand. Operating profits are $\Pi_z = \big(1 - \frac{w/q_z}{p_z}\big)p_z y_z = \big(1-\psi_z^{-1}\big)\delta_z Y$. Inverting Eq. (IA.5) gives $\delta_z = \frac{\zeta\psi_z - 1}{\zeta(\psi_z-1)}$, so scaled profits $\pi_z = \Pi_z/Y$ satisfy

$$\pi_z = \left(1 - \frac{1}{\psi_z}\right)\delta_z = 1 - \frac{1}{\zeta\psi_z}. \tag{IA.7}$$

Production labor is revenue divided by the markup and the wage, so labor demand is $l_z = \frac{\delta_z}{\psi_z\,\omega} = \frac{\pi_z}{(\psi_z-1)\,\omega}$, where $\omega = w/Y$ is the scaled wage. Writing m_s for the relative price when $q_L/q_F = \lambda^s$ and substituting Eq. (IA.3), profits are

$$\pi(s) = \frac{1-\zeta}{m_s^{\frac{\zeta}{1-\zeta}} + 1 - \zeta}, \qquad \pi(-s) = \frac{1-\zeta}{m_s^{-\frac{\zeta}{1-\zeta}} + 1 - \zeta}, \qquad \pi(0) = \frac{1-\zeta}{2-\zeta}, \tag{IA.8}$$

the corresponding gross markups are

$$\psi(s) = \frac{1+(1-\zeta)m_s^{-\frac{\zeta}{1-\zeta}}}{\zeta}, \qquad \psi(-s) = \frac{1+(1-\zeta)m_s^{\frac{\zeta}{1-\zeta}}}{\zeta}, \qquad \psi(0) = \frac{2-\zeta}{\zeta}, \tag{IA.9}$$

and labor demands are

$$l(s) = \frac{\zeta\, m_s^{\frac{\zeta}{1-\zeta}}}{\omega\left(1+m_s^{\frac{\zeta}{1-\zeta}}\right)\left(m_s^{\frac{\zeta}{1-\zeta}}+1-\zeta\right)}, \qquad l(-s) = \frac{\zeta\, m_s^{\frac{\zeta}{1-\zeta}}}{\omega\left(1+m_s^{\frac{\zeta}{1-\zeta}}\right)\left(1+(1-\zeta)m_s^{\frac{\zeta}{1-\zeta}}\right)},$$
$$l(0) = \frac{\zeta}{2\,\omega\,(2-\zeta)}. \tag{IA.10}$$

The next result establishes the properties of m_s asserted in Section 1.

Lemma IA.1 (Relative price: existence, uniqueness, and monotonicity) *For every $\frac{q_F}{q_L} \in (0,1]$, Eq. (IA.6) has a unique solution $m \in (0,1]$, and m is strictly increasing in $\frac{q_F}{q_L}$. Hence m_s is strictly decreasing in the technology gap s, and profits $\pi(\sigma)$ and markups $\psi(\sigma)$ are strictly increasing in the technology position σ.*

Proof. Rearranging Eq. (IA.6), m solves $h(m) = \frac{q_F}{q_L}$, where

$$h(m) \equiv m\,\frac{1+(1-\zeta)m^{\frac{\zeta}{1-\zeta}}}{1+(1-\zeta)m^{-\frac{\zeta}{1-\zeta}}}.$$

Taking logs and differentiating,

$$\frac{d\ln h}{d\ln m} = 1 + \frac{\zeta\, m^{\frac{\zeta}{1-\zeta}}}{1+(1-\zeta)m^{\frac{\zeta}{1-\zeta}}} + \frac{\zeta\, m^{-\frac{\zeta}{1-\zeta}}}{1+(1-\zeta)m^{-\frac{\zeta}{1-\zeta}}} > 0.$$

Thus, h is continuous and strictly increasing, with $h(1) = 1$ and, for $m \leq 1$, $h(m) \leq (2-\zeta)m \to 0$ as $m \to 0$. Therefore a unique solution $m \in (0,1]$ exists for each $\frac{q_F}{q_L} \in (0,1]$, and $m = h^{-1}\left(\frac{q_F}{q_L}\right)$ is strictly increasing. Setting $\frac{q_F}{q_L} = \lambda^{-s}$, which is strictly decreasing in s, shows that m_s is strictly decreasing in s.

For markups, differentiating the expressions above Eq. (IA.6):

$$\frac{d\psi_L}{dm} = -m^{-\frac{1}{1-\zeta}} < 0 \qquad \text{and} \qquad \frac{d\psi_F}{dm} = m^{\frac{2\zeta-1}{1-\zeta}} > 0.$$

As s rises, m_s falls, so $\psi(s)$ increases and $\psi(-s)$ declines. Moreover, for any $s > 0$, $m_s < 1$, so $\psi(s) > \psi(0) > \psi(-s)$. Thus markups are increasing in σ. Profits inherit the same monotonicity from Eq. (IA.7), because π_z is increasing in ψ_z. Hence $\pi(s) > \pi(0) > \pi(-s)$ for $s > 0$.

With perfect substitutes ($\zeta = 1$), Bertrand competition yields the familiar limit-pricing outcome: if $q_L > q_F$, the leader serves the entire market at a price equal to the laggard's marginal cost, earning gross markup $\frac{q_L}{q_F}$ and scaled profit $1 - \frac{q_F}{q_L}$, while the laggard earns zero profit. If $q_L = q_F$, both firms price at marginal cost and earn zero profits. The next result verifies the claim in Section 1 that the imperfect-substitutes expressions converge to these outcomes as $\zeta \to 1$.

Lemma IA.2 (Perfect-substitutes limit) *Fix a gap $s \geq 1$ and a scaled wage ω, and let $\zeta \to 1$. Then $m_s \to 1$ while $(1-\zeta)m_s^{-\frac{\zeta}{1-\zeta}} \to \lambda^s - 1$, and*

$$\psi(s) \to \lambda^s, \quad \psi(-s) \to 1, \quad \pi(s) \to 1 - \lambda^{-s}, \quad \pi(-s) \to 0, \quad \omega\, l(s) \to \lambda^{-s}, \quad \omega\, l(-s) \to 0.$$

For tied industries, $\psi(0) \to 1$, $\pi(0) \to 0$, and the CES limit is the symmetric allocation, so $\omega l(0) \to \frac{1}{2}$. These are the perfect-substitutes Bertrand outcomes, with limit pricing for $s \geq 1$.

Proof. Let $u \equiv (1-\zeta)m_s^{-\frac{\zeta}{1-\zeta}}$. Multiplying Eq. (IA.6) through by its denominator gives

$$m_s\left(1 + (1-\zeta)m_s^{\frac{\zeta}{1-\zeta}}\right) = \lambda^{-s}(1+u). \tag{IA.11}$$

Because $m_s \in (0,1]$ by Lemma IA.1, the left side of Eq. (IA.11) lies between m_s and $2-\zeta$, so

$$\lambda^s m_s - 1 \;\leq\; u \;\leq\; (2-\zeta)\lambda^s - 1 \;<\; 2\lambda^s.$$

Taking logs in the definition of u, the upper bound implies

$$|\ln m_s| = \frac{1-\zeta}{\zeta}\,\ln\frac{u}{1-\zeta} \;\leq\; \frac{1-\zeta}{\zeta}\,\ln\frac{2\lambda^s}{1-\zeta} \;\longrightarrow\; 0,$$

so $m_s \to 1$. The lower bound then gives $\liminf u \geq \lambda^s - 1 > 0$, so u is bounded away from zero along the limit and $m_s^{\frac{\zeta}{1-\zeta}} = \frac{1-\zeta}{u} \to 0$. Eq. (IA.11) then yields $u \to \lambda^s - 1$.

The stated limits then follow. For markups, $\psi(s) = \frac{1+u}{\zeta} \to \lambda^s$ and $\psi(-s) = \frac{1+(1-\zeta)m_s^{\zeta/(1-\zeta)}}{\zeta} \to 1$. For profits, substituting $m_s^{\frac{\zeta}{1-\zeta}} = \frac{1-\zeta}{u}$ into Eq. (IA.8) gives $\pi(s) = \frac{u}{1+u} \to 1 - \lambda^{-s}$ and $\pi(-s) = \frac{(1-\zeta)^2}{u+(1-\zeta)^2} \to 0$. For labor, the same substitution in Eq. (IA.10) gives $\omega\, l(s) = \frac{\zeta}{\left(1+m_s^{\zeta/(1-\zeta)}\right)(1+u)} \to \lambda^{-s}$ and $\omega\, l(-s) \to 0$. For tied industries, the limits follow directly from $\psi(0) = \frac{2-\zeta}{\zeta}$, $\pi(0) = \frac{1-\zeta}{2-\zeta}$, and $\omega l(0) = \frac{\zeta}{2(2-\zeta)}$. Thus $\psi(0) \to 1$, $\pi(0) \to 0$, and $\omega l(0) \to \frac{1}{2}$.

C.2 Entrepreneur problem, risk-adjusted returns, and proofs

This subsection summarizes the entrepreneur's portfolio and consumption problem. Given prices, entrepreneur i chooses consumption and the portfolio share $\xi_t = k_t^i/n_t^i$ invested in innovative firms (the next lemma shows that this share is independent of i). Net worth evolves as

$$\frac{dn_t^i}{n_t^i} = \left[r_t^f + \xi_t(r_t - r_t^f) - c_t\right] dt + \xi_t \sigma_{A,t} dZ_t^i, \tag{IA.12}$$

where $c_t = c_t^i/n_t^i$ is the consumption-wealth ratio.

Lemma IA.3 (Entrepreneurial policies) *Given prices, optimal consumption, investment in innovative firms, and bond holdings are linear in net worth:*

$$c_t^i = c_t n_t^i, \qquad k_t^i = \xi_t n_t^i, \qquad b_t^i = (1-\xi_t) n_t^i. \tag{IA.13}$$

Under the assumption that $r_t \geq r_t^f$, the portfolio share satisfies $\xi_t = \frac{r_t - r_t^f}{\nu \sigma_{A,t}^2}$. The consumption-wealth ratio satisfies

$$\frac{\dot{c}_t}{c_t} = c_t + (\epsilon - 1)\widehat{r}_t(\xi_t) - \epsilon\rho, \qquad \widehat{r}_t(\xi_t) = r_t^f + \xi_t(r_t - r_t^f) - \frac{1}{2}\nu\xi_t^2\sigma_{A,t}^2. \tag{IA.14}$$

Aggregate entrepreneurial consumption growth is

$$g_t^E = \epsilon\left(\widehat{r}_t(\xi_t) - \rho\right) + \frac{1}{2}\nu\xi_t^2\sigma_{A,t}^2. \tag{IA.15}$$

Proof. The budget constraint is linear in net worth and preferences are homothetic, so the value function is homogeneous and the optimal policies are linear in net worth. For any portfolio share ξ, the risky component of net-worth growth has instantaneous volatility $\xi\sigma_{A,t}$. Under relative risk aversion ν, the instantaneous risk-adjusted (certainty-equivalent) return on savings is therefore given by the second equation in Eq. (IA.14). Maximizing this risk-adjusted return gives the above expression for the portfolio share. Because the entrepreneur problem is homothetic, write $c_t^i = c_t n_t^i$ and conjecture

$$J(n,t) = \frac{(a_t n)^{1-\nu}}{1-\nu}.$$

The consumption FOC from the Duffie–Epstein HJB implies

$$c_t = \rho^\epsilon a_t^{1-\epsilon}.$$

Substituting the value-function conjecture and this FOC into the HJB gives

$$0 = \frac{\dot{a}_t}{a_t} + \widehat{r}_t(\xi_t) - c_t + \frac{c_t - \rho}{1 - 1/\epsilon}.$$

Combining these two equations yields

$$\frac{\dot{c}_t}{c_t} = c_t + (\epsilon - 1)\widehat{r}_t(\xi_t) - \epsilon\rho.$$

Aggregate entrepreneurial net worth grows at rate $r_t^f + \xi_t(r_t - r_t^f) - c_t$ because idiosyncratic risk washes out. Because $C_t^E = c_t \int n_t^i \, di$, aggregate entrepreneurial consumption growth is $g_t^E = \dot{c}_t/c_t + r_t^f + \xi_t(r_t - r_t^f) - c_t$. Combining this expression with Eq. (IA.14) and $r_t^f + \xi_t(r_t - r_t^f) = \widehat{r}_t(\xi_t) + \frac{1}{2}\nu\xi_t^2\sigma_{A,t}^2$ gives Eq. (IA.15).

We now use this lemma to prove Propositions 1 and 2. The idiosyncratic-risk premium

$$r - r^f = \nu\sigma_A^2 \tag{IA.16}$$

follows from Lemma IA.3 and bond-market clearing ($\xi_t = 1$). For the financing wedge, the firm's scaled Hamilton–Jacobi–Bellman equation along a BGP is Eq. (25). Using the definition of $\chi(\sigma)$ in Section 2 and the position-specific hurdle rate in Eq. (17), we have

$$r^k(\sigma) - g = (r - g)(1 + \chi(\sigma)),$$

so

$$r^k(\sigma) - r = (r - g)\chi(\sigma). \tag{IA.17}$$

Moreover, $\chi(\sigma) \geq 0$: Eq. (10) sets $x(\sigma)$ weakly below the unconstrained optimum $(G')^{-1}(\Delta v(\sigma)/\omega)$, and G' is increasing, so $\omega G'(x(\sigma)) \leq \Delta v(\sigma)$, with equality when the financing constraint is slack. Combining Eqs. (IA.16) and (IA.17) with $r > g$ then proves Proposition 1.

For the proof of Proposition 2, the aggregate entrepreneurial-consumption growth equation is Eq. (IA.15). In equilibrium, $\xi = 1$ and $\widehat{r} = r - \frac{1}{2}\nu\sigma_A^2$. Along a BGP, $g^E = g$, so

$$g = \epsilon\left(r - \frac{1}{2}\nu\sigma_A^2 - \rho\right) + \frac{1}{2}\nu\sigma_A^2. \tag{IA.18}$$

Solving for r gives the second expression in Eq. (23). Subtracting the risk premium $r - r^f = \nu\sigma_A^2$ gives the first expression there. The derivative comparison in Eq. (24) follows immediately.

Lemma IA.4 (Macroeconomic dynamics) *For a given constant level of idiosyncratic risk σ_A, the path of aggregate quantities, the labor share, innovation rates, the technology-gap distribution, and entrepreneurs' required return is the same as in a model without idiosyncratic risk but with time-preference rate*

$$\bar{\rho} = \rho + \frac{1}{2}\left(1 - \frac{1}{\epsilon}\right)\nu\sigma_A^2. \tag{IA.19}$$

However, with idiosyncratic risk, the financial cost of capital is lower than entrepreneurs' required return by $\nu\sigma_A^2$.

Proof. With bond-market clearing, $\xi_t = 1$ and $\widehat{r}_t = r_t - \frac{1}{2}\nu\sigma_A^2$, so Eq. (IA.14) becomes $\dot{c}_t/c_t = c_t + (\epsilon - 1)r_t - \epsilon\bar{\rho}$ with $\bar{\rho}$ in Eq. (IA.19), and aggregate entrepreneurial net worth grows at rate $r_t - c_t$. These are the consumption-wealth dynamics of an economy without idiosyncratic risk and with time-preference rate $\bar{\rho}$. Idiosyncratic risk enters the remaining equilibrium conditions—the static Bertrand block, the firm valuation equation, the gap-distribution dynamics, and market clearing (Internet Appendix C.5)—only through r_t. The two economies therefore share the same dynamic system for aggregate quantities, the labor share, innovation rates, the gap distribution, and r_t, and hence the same paths given the same initial conditions. The financial cost of capital differs because diversified claims are not exposed to the idiosyncratic diffusion risk: bond-market clearing requires $r_t^f = r_t - \nu\sigma_A^2$.

C.3 Partial diversification and uninsured idiosyncratic risk

This subsection provides a foundation for the uninsured idiosyncratic risk in the main text. We show that allowing entrepreneurs to sell and diversify only a limited fraction of their idiosyncratic exposure—a limit that can reflect incentive provision or other frictions—is equivalent to the main-text formulation in which raw idiosyncratic risk $\tilde{\sigma}_A$ is transformed into uninsured idiosyncratic risk $\sigma_A = \Lambda\tilde{\sigma}_A$, where Λ reflects the sensitivity to idiosyncratic performance.

Our analysis here is for fixed prices r_t and r_t^f. Entrepreneur i's net worth is $n_t^i = k_t^i + b_t^i$, where k_t^i is investment in entrepreneurial capital and b_t^i is the risk-free bond position. Suppose that, before any risk sharing, entrepreneurial capital exposes the entrepreneur to raw idiosyncratic risk $\widetilde{\sigma}_A$. The raw payoff flow on entrepreneurial capital is

$$d\widetilde{\Pi}_t^i = r_t k_t^i \, dt + \widetilde{\sigma}_A k_t^i \, dZ_t^i,$$

where Z_t^i is a standard Brownian motion, independent across entrepreneurs. The entrepreneur can sell a fraction ℓ_t^i subject to

$$0 \leq \ell_t^i \leq 1 - \Lambda, \qquad \Lambda \in (0, 1).$$

The sale proceeds are invested in a diversified pool of similar claims sold by other entrepreneurs. This pool diversifies idiosyncratic risks across the continuum of entrepreneurs. Thus, for each unit of entrepreneurial capital whose idiosyncratic exposure is sold, the entrepreneur gives up the corresponding idiosyncratic return component and receives the diversified payoff associated with the same claim. The transaction therefore leaves unaffected the entrepreneur's date-t net worth and the deterministic component of the return. However, the idiosyncratic exposure borne by the entrepreneur is reduced.[42]

Given sale share ℓ_t^i, entrepreneur i's payoff from entrepreneurial capital is the retained payoff from own capital plus the payoff on the diversified pool:

$$d\Pi_t^i(\ell_t^i) = (1-\ell_t^i)\left(r_t k_t^i\,dt + \widetilde{\sigma}_A k_t^i\,dZ_t^i\right) + \ell_t^i r_t k_t^i\,dt$$
$$= r_t k_t^i\,dt + (1-\ell_t^i)\widetilde{\sigma}_A k_t^i\,dZ_t^i. \tag{IA.20}$$

The deterministic component is unchanged because the sold claim is replaced by a diversified claim with the same deterministic payoff. The idiosyncratic exposure is reduced from $\widetilde{\sigma}_A$ to $(1-\ell_t^i)\widetilde{\sigma}_A$.

The entrepreneur's flow budget constraint can therefore be written as

$$dn_t^i = \left[r_t k_t^i + r_t^f b_t^i - c_t^i\right]dt + (1-\ell_t^i)\widetilde{\sigma}_A k_t^i\,dZ_t^i, \qquad n_t^i = k_t^i + b_t^i, \tag{IA.21}$$

with the constraint $0 \leq \ell_t^i \leq 1-\Lambda$.

Because the sale and purchase of diversified claims leave the deterministic return and the budget set unchanged while reducing the entrepreneur's idiosyncratic exposure, a risk-averse entrepreneur chooses the maximum feasible amount of diversification. Specifically, let $\xi_t^i \equiv k_t^i/n_t^i$ denote the share of net worth invested in entrepreneurial capital. For a given ξ_t^i, the instantaneous risk-adjusted return on saving is

$$\widehat{r}_t(\xi_t^i,\ell_t^i) = r_t^f + \xi_t^i(r_t - r_t^f) - \frac{1}{2}\nu(\xi_t^i)^2(1-\ell_t^i)^2\widetilde{\sigma}_A^2. \tag{IA.22}$$

This expression is strictly increasing in ℓ_t^i whenever $\xi_t^i > 0$, $\nu > 0$, and $\widetilde{\sigma}_A > 0$. Hence,

$$\ell_t^i = 1-\Lambda.$$

Substituting this optimal issuance into Eq. (IA.21) gives

$$dn_t^i = \left[r_t k_t^i + r_t^f b_t^i - c_t^i\right]dt + \Lambda\widetilde{\sigma}_A k_t^i\,dZ_t^i. \tag{IA.23}$$

Thus the entrepreneur problem is equivalent to the main-text formulation with residual uninsured idiosyncratic volatility

$$\sigma_A = \Lambda\widetilde{\sigma}_A. \tag{IA.24}$$

Overall, selling and diversifying a fraction of the idiosyncratic profit flow scales the idiosyncratic component of entrepreneurial returns, while leaving the deterministic component unchanged. This is one foundation for working directly with residual uninsured risk σ_A in the main text.

[42] Equivalently, this is a mutual risk-sharing arrangement for the idiosyncratic component of entrepreneurial profits. The proceeds from selling claims are immediately used to purchase claims in the diversified pool, so the portfolio identity remains $n_t^i = k_t^i + b_t^i$. The arrangement is limited by $\ell_t^i \leq 1-\Lambda$ and cannot be used to create additional net positions in risk-free assets. The transactions can also be modeled as a transfer payment to the entrepreneur with $dT_t^i = -(1-\Lambda)\tilde{\sigma}_A k_t^i dZ_t^i$ (Angeletos and Panousi 2011).

C.4 Idiosyncratic investment risk and creative destruction

Internet Appendix C.3 showed how allowing entrepreneurs to sell and diversify a limited fraction of their idiosyncratic exposure is equivalent to the main-text formulation in which raw idiosyncratic risk is transformed into uninsured idiosyncratic risk. This subsection turns to entrepreneurs' portfolios of claims on firms and describes how idiosyncratic investment risk can be present without directly altering the model's creative-destruction dynamics. (Idiosyncratic risk still shapes creative destruction indirectly—through the endogenous required return on entrepreneurial capital and firms' innovation strategies—an interaction that is central to the paper.) The key mechanism is a reshuffling of ownership claims. Let $\mathbb{F}$ denote the set of firms, and let $a \in \mathbb{F}$ index an individual firm. Let $\mathcal{V}_{a,t}$ denote the value of firm a at date t, discounting future profits using entrepreneurs' required return.

We focus on the entrepreneur's portfolio of (divisible) firm-level ownership and control claims. A claim on firm a pays the cash flows generated by that firm and carries associated control rights; the specific identity of claimholders does not affect firm-level policies (because of the homotheticity of preferences and the linearity of entrepreneurs' constraints, all entrepreneurs have the same consumption-wealth share and there is a common required return). Let $\varpi^i_{a,t}$ denote entrepreneur i's holdings of claims on firm a, with aggregate supply normalized to one for each firm. Holdings clear firm by firm:

$$\int_0^1 \varpi^i_{a,t}\, di = 1 \quad \text{for each } a \in \mathbb{F}. \tag{IA.25}$$

The value of entrepreneur i's portfolio of firm claims is

$$k^i_t = \int_{\mathbb{F}} \varpi^i_{a,t} \mathcal{V}_{a,t}\, da. \tag{IA.26}$$

Suppose organizational turnover reshuffles claims in a way that changes entrepreneur i's portfolio value by $\sigma_A k^i_t dZ^i_t$, where Z^i_t is a standard Brownian motion independent across entrepreneurs. The post-reshuffling holdings $\hat{\varpi}^i_{a,t}$ satisfy

$$\int_{\mathbb{F}} \hat{\varpi}^i_{a,t} \mathcal{V}_{a,t}\, da = k^i_t + \sigma_A k^i_t dZ^i_t, \quad \text{with} \quad \int_0^1 \hat{\varpi}^i_{a,t}\, di = 1 \quad \text{for each } a \in \mathbb{F}. \tag{IA.27}$$

The reshuffling changes who bears firm-level cash-flow risk but leaves firm technology states, pricing, labor demand, and innovation policies unchanged. The holdings $\hat{\varpi}^i_{a,t}$ are divisible financial claims to firm cash flows and are not restricted in sign, so the value targets in Eq. (IA.27) are always attainable; because the identity of claimholders does not affect firm-level policies, the sign of an individual position is immaterial and reflects only the redistribution of cash-flow risk across entrepreneurs. Integrating the first condition in Eq. (IA.27) across entrepreneurs and using firm-by-firm clearing gives $\int_0^1 k^i_t\, di = \int_{\mathbb{F}} \hat{\varpi}^i_{a,t} \mathcal{V}_{a,t}\, da\, di = \int_{\mathbb{F}} \mathcal{V}_{a,t}\, da$ both before and after the reshuffling, so aggregate claims equal aggregate firm value. Consistency thus requires $\int_0^1 \sigma_A k_t^i \, dZ_t^i \, di = 0$, which holds by the law of large numbers for the continuum of independent shocks $\{dZ_t^i\}$ (this is the same convention under which idiosyncratic uncertainty washes out in the aggregate elsewhere in the paper).

Let $d\mathcal{R}_{a,t}$ denote the payoff on a claim to firm a over dt, including operating profits net of R&D costs and capital gains. In equilibrium, the expected return on holding firm a is r_t, so $E_t[d\mathcal{R}_{a,t}] = r_t \mathcal{V}_{a,t} dt$. Because each entrepreneur holds a continuum of claims, firm-level (Poisson) innovation risk washes out within the portfolio. Entrepreneur i's capital income therefore satisfies

$$\begin{aligned} dk_t^i &= \int_{\mathbb{F}} \hat{\varpi}_{a,t}^i E_t[d\mathcal{R}_{a,t}] \, da + \sigma_A k_t^i dZ_t^i \\ &= r_t \int_{\mathbb{F}} \hat{\varpi}_{a,t}^i \mathcal{V}_{a,t} \, da \, dt + \sigma_A k_t^i dZ_t^i = r_t k_t^i dt + \sigma_A k_t^i dZ_t^i, \end{aligned}$$

which gives Eq. (11). This construction shows that entrepreneurs' firm-level portfolio holdings can be consistent with both uninsured idiosyncratic return risk and the model's creative-destruction dynamics.

C.5 Dynamic equilibrium and balanced growth path

This subsection formally defines a dynamic equilibrium and a balanced growth path (BGP) for the benchmark economy—the environment studied in the main text up to Section 7. We write the dynamic system allowing for a deterministic path of idiosyncratic risk $\{\sigma_{A,t}\}_{t\geq 0}$, which nests the constant-parameter benchmark and the transition analysis in Section 3; the BGP definition imposes constant parameters. We first state the definition in terms of prices, individual plans, and market clearing, consistent with the description in the main text (Section 1). We then reduce equilibrium to a system of equations, verify that the number of equations equals the number of endogenous variables, and explain how all remaining equilibrium variables are recovered. Finally, we define a BGP and verify the same correspondence.

Initial conditions and endowments. Let $\mathbb{F} = [0,1] \times \{1,2\}$ denote the set of firms, with $j \in [0,1]$ indexing industries and $z \in \{1,2\}$ indexing the two firms in each industry. The initial state is the productivity of each firm:

$$\mathcal{Q}_0 \equiv \{q_{z,j,0} : j \in [0,1], \; z \in \{1,2\}\}.$$

We assume the initial productivities to be compatible with the ladder: within each industry the two firms lie on a common quality ladder, so that $q_{1,j,0}/q_{2,j,0} = \lambda^{n_j}$ for some $n_j \in S$, which makes the initial gap $s_{j,0} = |n_j| \in S^+$ well defined. From this initial state, one can determine the initial leader productivity in each industry, $q_{L,j,0} \equiv$

$\max\{q_{1,j,0}, q_{2,j,0}\}$, the initial gap $s_{j,0} \in S^+$, the initial aggregate quality index

$$Q_0 \equiv \exp\left\{\int_0^1 \ln q_{L,j,0}\, dj\right\}, \tag{IA.28}$$

and the initial gap distribution

$$\mu_0(s) \equiv \int_0^1 \mathbb{1}\{s_{j,0} = s\}\, dj, \qquad s \in S^+. \tag{IA.29}$$

Firm a's technology position at date t is $\sigma_{a,t} \in S$, the number of rungs by which it leads (if positive) or trails (if negative) its industry rival. Workers form a unit measure, supply one unit of labor inelastically, and consume their wage income. Entrepreneurs $i \in [0,1]$ are collectively endowed at date 0 with the ownership claims to all firms and hold no bonds, so bonds are in zero net supply.

Definition (dynamic equilibrium). Given the initial technologies $\mathcal{Q}_0$ and the initial assignment of firm claims to entrepreneurs, a *dynamic equilibrium* consists of price paths $\{w_t, r_t, r_t^f\}_{t\geq 0}$; entrepreneurial plans $\{c_t^i, k_t^i, b_t^i\}_{t\geq 0}$ for $i \in [0,1]$; firm prices, production labor, innovation rates, and values $\{p_{a,t}, l_{a,t}, x_{a,t}, V_{a,t}\}_{t\geq 0}$ for $a \in \mathbb{F}$; aggregate quantities $\{Y_t, C_t^E, C_t^W, K_t, \mathcal{B}_t\}_{t\geq 0}$; and the technology-gap distribution $\{\mu_t(s)\}_{t\geq 0}$ for $s \in S^+$, such that:

(a) *Production and demand.* Output Y_t is given by the final-good technology in Eq. (3) with $y_{z,j,t} = q_{z,j,t} l_{z,j,t}$. The final-good market is competitive, so expenditure in each industry equals Y_t and within-industry relative demand satisfies $p_{1,j,t}/p_{2,j,t} = (y_{1,j,t}/y_{2,j,t})^{\zeta-1}$.

(b) *Intermediate-goods competition.* In each industry, given the wage w_t and the demand system in (a), the prices $(p_{1,j,t}, p_{2,j,t})$ constitute the static Bertrand equilibrium derived in Internet Appendix C.1, and $l_{a,t}$ is the implied production-labor demand.

(c) *Firm optimization and valuation.* Given the paths $\{Y_t, w_t, r_t\}_{t\geq 0}$ and a (position-indexed) competitor strategy $\{x_t^c(\sigma)\}_{\sigma\in S}$, the (position-indexed) value $V_t(\sigma)$ solves the Hamilton–Jacobi–Bellman equation (8) subject to the pledgeability constraint (9), the conventions $\Delta V_t(\bar{s}) = 0$, $\Delta^c V_t(-\bar{s}) = 0$, and $x_t(\bar{s}) = 0$, and the no-bubble condition $\lim_{T\to\infty} E_t\big[e^{-\int_t^T r_u\, du}\, V_T(\sigma_T)\big] = 0$, where σ_T denotes the firm's position process. The innovation policy $x_t(\sigma)$ attains the maximum in Eq. (8), and firm-level variables satisfy $x_{a,t} = x_t(\sigma_{a,t})$ and $V_{a,t} = V_t(\sigma_{a,t})$.

(d) *Symmetry.* Each firm's conjecture about its competitor coincides with the competitor's equilibrium strategy: $x_t^c(\sigma) = x_t(\sigma)$ for all $\sigma \in S$ and $t \geq 0$.

(e) *Entrepreneur optimization.* Given $\{r_t, r_t^f\}_{t\geq 0}$, each plan $\{c_t^i, k_t^i, b_t^i\}_{t\geq 0}$ maximizes lifetime utility (1) subject to the budget constraint (12), nonnegative consumption and firm holdings, and the natural borrowing limit.

(f) *Aggregation and consistency.* Aggregates are the cross-sectional integrals of individual plans, $C_t^E = \int_0^1 c_t^i \, di$, $K_t = \int_0^1 k_t^i \, di$, and $\mathcal{B}_t = \int_0^1 b_t^i \, di$, and workers consume their wage income, $C_t^W = w_t$. Firm technology positions evolve according to the quality-ladder dynamics of Section 1, with innovation arrival intensities $\{x_{a,t}\}$. Because arrivals are independent across industries, the gap distribution evolves deterministically according to Eq. (4), with initial condition (IA.29).

(g) *Market clearing.* At every t: the labor market clears, $\int_{\mathbb{F}} \left[l_{a,t} + G(x_{a,t})\right] da = 1$; the goods market clears, $C_t^E + C_t^W = Y_t$; the market for firm claims clears, $K_t = \int_{\mathbb{F}} V_{a,t} \, da$; and the bond market clears, $\mathcal{B}_t = 0$.

As usual, one market-clearing condition in (g) is redundant: given agents' budget constraints and the valuation equations in (c), clearing of the labor, bond, and firm-claims markets implies goods-market clearing (Walras's law). The reduced system below therefore imposes three independent market-clearing restrictions per date.

Reducing dimensionality. Two observations reduce the equilibrium to a finite-dimensional system at each date. First, by Lemma IA.3, entrepreneurs' optimal policies are linear in net worth with common coefficients, so aggregates do not depend on the cross-sectional distribution of entrepreneurial wealth. Second, in any symmetric equilibrium, all firms with the same technology position at date t have the same relative price, markup, production labor, innovation rate, scaled profit, and scaled value. If firm $a \in \mathbb{F}$ has technology position $\sigma_{a,t} = \sigma$, then

$$x_{a,t} = x_t(\sigma), \qquad \psi_{a,t} = \psi(\sigma), \qquad l_{a,t} = l_t(\sigma), \qquad V_{a,t} = Y_t v_t(\sigma). \tag{IA.30}$$

Raw prices are recovered from the static pricing rule $p_{a,t} = \psi(\sigma_{a,t}) w_t / q_{a,t}$, so they depend on absolute productivity (in addition to depending on relative productivity). Moreover, the static Bertrand block implies that revenue shares, markups, scaled profits, relative prices, and production labor depend only on the position and the scaled wage. In particular, production labor equals the firm's revenue divided by its gross markup and the wage; because within-industry revenue shares and markups depend only on the gap,

$$l_t(\sigma) = \frac{\tilde{l}(\sigma)}{\omega_t}, \tag{IA.31}$$

where $\tilde{l}(\sigma)$, the firm's within-industry revenue share divided by its gross markup $\psi(\sigma)$, is a function of σ alone (Internet Appendix C.1).

The equilibrium system. Given the initial condition $\mathcal{Q}_0$, equilibrium is characterized by a differentiable path

$$\mathcal{E} = \left\{ c_t, \omega_t, r_t, r_t^f, g_t^Y, \{x_t(\sigma), v_t(\sigma)\}_{\sigma \in S}, \{\mu_t(s)\}_{s \in S^+} \right\}_{t \geq 0}, \tag{IA.32}$$

where c_t is entrepreneurs' consumption-wealth ratio, ω_t is the scaled wage (the labor share), r_t is entrepreneurs' required return on capital, r_t^f is the financial cost of capital, and $g_t^Y \equiv \dot{Y}_t/Y_t$ is output growth. Conditions (i)–(viii) below, together with the boundary conditions that follow them, are satisfied by every dynamic equilibrium. A path $\mathcal{E}$ satisfying these conditions generates a dynamic equilibrium through the recovery mapping at the end of this subsection.

(i) *Portfolio choice and bond-market clearing.* By Lemma IA.3, all entrepreneurs choose the same portfolio share $\xi_t = (r_t - r_t^f)/(\nu\sigma_{A,t}^2)$ of net worth in firm claims, if $\sigma_{A,t} > 0$. (If $\sigma_{A,t} = 0$, then $r_t = r_t^f$.) Thus, aggregate bond holdings are $\mathcal{B}_t = (1 - \xi_t)N_t$, where $N_t = K_t + \mathcal{B}_t$ is aggregate net worth. Bond-market clearing with $N_t > 0$ therefore requires $\xi_t = 1$, that is,

$$r_t - r_t^f = \nu\sigma_{A,t}^2. \tag{IA.33}$$

(ii) *Entrepreneurs' consumption-wealth ratio.* With $\xi_t = 1$, the risk-adjusted return on entrepreneurial saving is $\widehat{r}_t = r_t - \frac{1}{2}\nu\sigma_{A,t}^2$, and Lemma IA.3 implies

$$\frac{\dot{c}_t}{c_t} = c_t + (\epsilon - 1)\widehat{r}_t - \epsilon\rho. \tag{IA.34}$$

(iii) *Firm values.* For every $\sigma \in S$, the scaled firm value satisfies the continuous-time HJB

$$(r_t - g_t^Y)v_t(\sigma) - \dot{v}_t(\sigma) = \pi(\sigma) - \omega_t G(x_t(\sigma)) + x_t(\sigma)\Delta v_t(\sigma) + x_t(-\sigma)\Delta^c v_t(\sigma), \tag{IA.35}$$

where $\Delta v_t(\sigma) \equiv \Delta V_t(\sigma)/Y_t$ and $\Delta^c v_t(\sigma) \equiv \Delta^c V_t(\sigma)/Y_t$ are the scaled capital gains from own and competitor innovation, with the conventions $\Delta v_t(\bar{s}) = 0$ and $\Delta^c v_t(-\bar{s}) = 0$. Symmetry, condition (d) of the definition, is imposed by writing the competitor's innovation rate as $x_t(-\sigma)$.

(iv) *Firm innovation.* For every $\sigma < \bar{s}$, innovation satisfies

$$x_t(\sigma) = \min\left\{(G')^{-1}\left(\frac{\Delta v_t(\sigma)}{\omega_t}\right), G^{-1}\left(\frac{\alpha v_t(\sigma)}{\omega_t}\right)\right\}, \tag{IA.36}$$

together with the boundary condition $x_t(\bar{s}) = 0$. The first term in the minimum is the unconstrained innovation rate; the second is the maximum rate consistent with the pledgeability constraint.

(v) *Technology-gap distribution.* For $s = 1, \ldots, \bar{s}$, the gap distribution evolves according to

$$\dot{\mu}_t(s) = \mu_t(s+1)(1-\phi)x_t(-(s+1)) + \mu_t(s-1)x_t(s-1)(1+\mathbb{1}_{s=1}) - \big(x_t(s) + x_t(-s)\big)\mu_t(s), \tag{IA.37}$$

with the convention $\mu_t(\bar{s}+1) = 0$, together with the normalization

$$\sum_{s \in S^+} \mu_t(s) = 1. \tag{IA.38}$$

(vi) *Labor-market clearing.* Production labor and R&D labor sum to the unit labor endowment:

$$\sum_{s \in S^+} \mu_t(s) \left[G(x_t(s)) + G(x_t(-s)) + l_t(s) + l_t(-s) \right] = 1. \tag{IA.39}$$

By Eq. (IA.31), production labor is decreasing in ω_t, so this condition determines the labor share given innovation rates and the gap distribution.

(vii) *Goods-market and firm-ownership clearing.* Workers consume their wage income ($C_t^W = w_t$), so goods-market clearing, $C_t^E + C_t^W = Y_t$, implies $C_t^E = (1-\omega_t)Y_t$. The linear consumption rule and bond-market clearing imply $C_t^E = c_t N_t = c_t K_t$, and clearing of the market for firm claims requires $K_t = \int_{\mathbb{F}} V_{a,t}\, da$. Dividing by Y_t and using Eq. (IA.30),

$$2\mu_t(0)v_t(0) + \sum_{s=1}^{\bar{s}} \mu_t(s)\big(v_t(s) + v_t(-s)\big) = \frac{1-\omega_t}{c_t}. \tag{IA.40}$$

The factor of two for $s = 0$ reflects that a tied industry has two firms with technology position $\sigma = 0$. Consistent with the Walras's-law remark above, the goods and firm-claims markets contribute a single independent restriction per date, captured by Eq. (IA.40). Ownership clearing at date 0 holds by the endowment of firm claims.

(viii) *Output growth.* The quality index grows according to

$$\frac{\dot{Q}_t}{Q_t} = \ln\lambda \sum_{s \in S^+} \mu_t(s)(1 + \mathbb{1}_{s=0})x_t(s), \tag{IA.41}$$

and output is determined by the quality index, the gap distribution, and the static labor allocation:

$$\ln Y_t = \ln Q_t + \sum_{s \in S^+} \mu_t(s) \ln \left[l_t(s)^\zeta + \left(\lambda^{-s} l_t(-s)\right)^\zeta \right]^{1/\zeta}. \tag{IA.42}$$

Substituting Eq. (IA.31) and using the normalization (IA.38), Eq. (IA.42) can be written as

$$\ln Y_t = \ln Q_t - \ln\omega_t + \sum_{s \in S^+} \mu_t(s)\Phi(s), \qquad \Phi(s) \equiv \frac{1}{\zeta} \ln\Big[\tilde{l}(s)^\zeta + \left(\lambda^{-s}\tilde{l}(-s)\right)^\zeta \Big]. \tag{IA.43}$$

Equations (IA.41) and (IA.43) determine Q_t and Y_t. Differentiating Eq. (IA.43) with respect to time:

$$g_t^Y = \ln\lambda \sum_{s \in S^+} \mu_t(s)(1 + \mathbb{1}_{s=0})x_t(s) - \frac{\dot{\omega}_t}{\omega_t} + \sum_{s \in S^+} \dot{\mu}_t(s)\Phi(s), \tag{IA.44}$$

where $\dot{\mu}_t(s)$ for $s \geq 1$ is given by Eq. (IA.37) and $\dot{\mu}_t(0) = -\sum_{s=1}^{\bar{s}} \dot{\mu}_t(s)$.

Boundary conditions. Conditions (ii), (iii), (v), and (viii) involve the time derivatives $\dot{c}_t$, $\dot{v}_t(\sigma)$, $\dot{\mu}_t(s)$, and $\dot{\omega}_t$; the remaining conditions are static. The gap shares have initial values given by Eq. (IA.29). The consumption-wealth ratio and the scaled firm values are forward-looking variables. Their paths must satisfy entrepreneurs' transversality condition and the no-bubble condition for firm valuation. For the transition analysis, in which parameters are eventually constant, we impose these forward-looking restrictions by requiring c_t and $\{v_t(\sigma)\}_{\sigma\in S}$ to converge to the terminal balanced growth path defined below. This provides $2\bar{s}+2$ terminal or transversality restrictions—one for c_t and one for each $v_t(\sigma)$, $\sigma \in S$. Finally, note that $\dot{\omega}_t$ enters only through the output-growth condition (viii). Since ω_t is pinned down by the labor-market clearing condition at each date, $\dot{\omega}_t$ is simply the derivative of that path and requires no additional boundary condition.

Counting equations and endogenous variables. At each date t, the path in Eq. (IA.32) lists $5\bar{s}+8$ endogenous variables: five scalars $(c_t, \omega_t, r_t, r_t^f, g_t^Y)$, $2\bar{s}+1$ innovation rates $\{x_t(\sigma)\}_{\sigma\in S}$, $2\bar{s}+1$ scaled firm values $\{v_t(\sigma)\}_{\sigma\in S}$, and $\bar{s}+1$ gap shares $\{\mu_t(s)\}_{s\in S^+}$, so that $5+(2\bar{s}+1)+(2\bar{s}+1)+(\bar{s}+1)=5\bar{s}+8$. Conditions (i)–(viii) provide the same number of equations at each date:

$$\underbrace{1}_{\text{(i)}}+\underbrace{1}_{\text{(ii)}}+\underbrace{2\bar{s}+1}_{\text{(iii)}}+\underbrace{2\bar{s}+1}_{\text{(iv)}}+\underbrace{\bar{s}+1}_{\text{(v)}}+\underbrace{1}_{\text{(vi)}}+\underbrace{1}_{\text{(vii)}}+\underbrace{1}_{\text{(viii)}}=5\bar{s}+8,$$

where (iv) counts the $2\bar{s}$ first-order conditions and the boundary condition $x_t(\bar{s})=0$, and (v) counts the $\bar{s}$ flow equations and the normalization. The correspondence is block by block: the value equations (iii) match the scaled values, the innovation conditions (iv) match the innovation rates, the distribution conditions (v) match the gap shares, and the five scalar conditions match the five scalars—heuristically, (i) determines r_t^f given r_t, (ii) is the differential equation for c_t, (vi) determines ω_t, (vii) disciplines r_t, and (viii) determines g_t^Y.

Recovering the remaining equilibrium variables. All other equilibrium variables follow from $\mathcal{E}$. The aggregate quality index Q_t follows from Eq. (IA.41) and the initial condition (IA.28), and output Y_t follows from Eq. (IA.43). The wage and worker consumption satisfy $C_t^W = w_t = \omega_t Y_t$, entrepreneurial consumption is $C_t^E = (1-\omega_t)Y_t$, aggregate holdings of firm claims is $K_t = C_t^E/c_t$, and bond holdings are $\mathcal{B}_t = 0$. Individual plans follow from the linear policy rules of Lemma IA.3 with $\xi_t = 1$, given initial net worths equal to the value of each entrepreneur's endowed claims. Firm positions $\sigma_{a,t}$ evolve according to the innovation arrivals. Firm-level prices, markups, profits, and production labor are recovered from the static Bertrand system and Eq. (IA.31). Innovation rates and values are recovered from $x_{a,t} = x_t(\sigma_{a,t})$ and $V_{a,t} = Y_t v_t(\sigma_{a,t})$. By construction, the recovered variables satisfy requirements (a)–(g) of the definition.

Balanced growth path. A *balanced growth path* (BGP) is a dynamic equilibrium with constant parameter values in which the technology-gap distribution is stationary; the consumption-wealth ratio, labor share, required return, financial cost of capital, and the (position-indexed) innovation rates and scaled firm values are constant; and aggregate output, aggregate quality, the wage, entrepreneurial consumption, worker consumption, and the total value of firms grow at the common constant rate $g = g_t^Y$. We represent a BGP with the tuple

$$\mathcal{E}^{BGP} = \left(c, g, \omega, r, r^f, \{x(\sigma), v(\sigma)\}_{\sigma\in S}, \{\mu(s)\}_{s\in S^+}\right), \tag{IA.45}$$

with $r > g$, so that scaled firm values are finite. Setting $\dot{c}_t = \dot{v}_t(\sigma) = \dot{\mu}_t(s) = \dot{\omega}_t = 0$ and $g_t^Y = g$ in conditions (i)–(viii) yields the BGP system.

(i) *Risk premium.* The financial cost of capital satisfies

$$r - r^f = \nu\sigma_A^2. \tag{IA.46}$$

(ii) *Entrepreneurs' consumption-wealth ratio.* The stationary version of Eq. (IA.34) is

$$0 = c + (\epsilon - 1)\left(r - \frac{1}{2}\nu\sigma_A^2\right) - \epsilon\rho. \tag{IA.47}$$

(iii) *Firm values.* For every $\sigma \in S$, the scaled firm value satisfies

$$(r-g)v(\sigma) = \pi(\sigma) - \omega G(x(\sigma)) + x(\sigma)\Delta v(\sigma) + x(-\sigma)\Delta^c v(\sigma), \tag{IA.48}$$

with the conventions $\Delta v(\bar{s}) = 0$ and $\Delta^c v(-\bar{s}) = 0$.

(iv) *Firm innovation.* For every $\sigma < \bar{s}$,

$$x(\sigma) = \min\left\{(G')^{-1}\left(\frac{\Delta v(\sigma)}{\omega}\right), G^{-1}\left(\frac{\alpha v(\sigma)}{\omega}\right)\right\}, \tag{IA.49}$$

together with the boundary condition $x(\bar{s}) = 0$.

(v) *Stationary gap distribution.* For $s = 1, \ldots, \bar{s}$,

$$0 = \mu(s+1)(1-\phi)x(-(s+1)) + \mu(s-1)x(s-1)(1+\mathbb{1}_{s=1}) - \big(x(s)+x(-s)\big)\mu(s), \tag{IA.50}$$

with the convention $\mu(\bar{s}+1) = 0$, together with the normalization

$$\sum_{s\in S^+}\mu(s) = 1. \tag{IA.51}$$

(vi) *Labor-market clearing.* Production labor and R&D labor sum to the unit labor endowment:

$$\sum_{s\in S^+}\mu(s)\left[G(x(s)) + G(x(-s)) + l(s) + l(-s)\right] = 1, \qquad l(\sigma) = \frac{\tilde{l}(\sigma)}{\omega}. \tag{IA.52}$$

(vii) *Goods-market and firm-ownership clearing.* The stationary version of Eq. (IA.40) is

$$2\mu(0)v(0) + \sum_{s=1}^{\bar{s}}\mu(s)\big(v(s) + v(-s)\big) = \frac{1-\omega}{c}. \tag{IA.53}$$

(viii) *Aggregate growth.* With $\dot{\mu}(s) = \dot{\omega} = 0$, Eq. (IA.44) reduces to

$$g = \ln \lambda \sum_{s \in S^+} \mu(s)(1 + \mathbb{1}_{s=0})x(s), \tag{IA.54}$$

which coincides with Eq. (15) in the main text: on a BGP, output growth equals quality growth.

Counting equations and endogenous variables for the BGP. The tuple in Eq. (IA.45) contains $5\bar{s} + 8$ endogenous variables: five scalars (c, g, ω, r, r^f), $2\bar{s} + 1$ innovation rates, $2\bar{s} + 1$ scaled firm values, and $\bar{s} + 1$ gap shares. Because the BGP conditions are the date-by-date conditions (i)–(viii) with time derivatives set to zero, the count of equations is identical to the dynamic case:

$$\underbrace{1}_{\text{(i)}} + \underbrace{1}_{\text{(ii)}} + \underbrace{2\bar{s} + 1}_{\text{(iii)}} + \underbrace{2\bar{s} + 1}_{\text{(iv)}} + \underbrace{\bar{s} + 1}_{\text{(v)}} + \underbrace{1}_{\text{(vi)}} + \underbrace{1}_{\text{(vii)}} + \underbrace{1}_{\text{(viii)}} = 5\bar{s} + 8.$$

Connection to the main-text return expressions and the BGP solution method. The expressions for entrepreneurs' required return and the financial cost of capital in Proposition 2 follow from this system. To see this, aggregate Eq. (IA.48) across firms, weighting firms in position s and $-s$ by $\mu(s)$ and position 0 by $2\mu(0)$. The own- and competitor-innovation terms combine into the change in aggregate scaled firm value induced by the flows of industries across gap states. Under a stationary distribution $\{\mu(s)\}_{s \in S^+}$, this change in aggregate scaled firm value is zero. Each industry's operating profits sum to one minus the industry's scaled production wage bill, $\pi(s) + \pi(-s) = 1 - \omega\big(l(s) + l(-s)\big)$. Using labor-market clearing (IA.52) to combine production and R&D labor then gives

$$(r - g)\left[2\mu(0)v(0) + \sum_{s=1}^{\bar{s}} \mu(s)\big(v(s) + v(-s)\big)\right] = 1 - \omega.$$

Combining this identity with Eq. (IA.53) gives $c = r - g$; note that the requirement $r > g$ is thus equivalent to a positive consumption-wealth ratio. Substituting $c = r - g$ into Eq. (IA.47) yields

$$r = \rho + \frac{g}{\epsilon} + \frac{1}{2}\left(1 - \frac{1}{\epsilon}\right)\nu\sigma_A^2,$$

and Eq. (IA.46) then gives

$$r^f = \rho + \frac{g}{\epsilon} - \frac{1}{2}\left(1 + \frac{1}{\epsilon}\right)\nu\sigma_A^2,$$

the expressions in Eq. (23). These closed forms underlie the computational procedure in Section 2: given a conjecture for (g, ω), the required return is $r = \rho + g/\epsilon + \frac{1}{2}(1 - 1/\epsilon)\nu\sigma_A^2$; Eqs. (IA.48)–(IA.51) then determine (v, x, μ); and the conjecture is updated until the growth and labor-market-clearing conditions, Eqs. (IA.54) and (IA.52), hold. The remaining variables (c, r^f) follow from $c = r - g$ and Eq. (IA.46).

D. Data description and sources

Markup distribution. In our benchmark analysis, for the markup distribution, we obtain data values from Hall (2018), which estimates the distribution of Lerner indexes. The Lerner index is the ratio of price minus marginal cost to price. To obtain markup moment values, we make many draws from the distribution of Lerner indexes in Hall (2018) and convert each Lerner index draw to a markup. The mean, median, and 90$^{\text{th}}$ percentile of the Lerner index in Hall (2018) are 15%, 12%, and 30%. The corresponding Lerner-index moments in our model are 15%, 12%, and 29%, indicating a very good fit.

Innovation output, profit volatility, and R&D-to-sales. Three types of moments are calculated at least in part using CRSP/Compustat merged data: innovation output, profit volatility, and R&D intensity. The data frequency is annual (except for the use of quarterly data to calculate the number of quarters with positive profits). Model moments are computed from simulations of 1,100 industries over 60 years. The model is solved at a monthly frequency and simulated monthly; firm-level variables are then aggregated to annual frequency to match the data. To focus on innovative firms, data moments are calculated for firm-years with positive R&D. Almost all firms have positive R&D in the model, due to the Inada-type condition that $G'(0) = 0$. More specifically, all firms with $\sigma < \bar{s}$ have strictly positive R&D, and the mass of firms with $\sigma = \bar{s}$, the maximum technological advantage, is very low in the calibrated model. Profit growth, profit volatility, and R&D-to-sales are calculated for a consistent sample so that all variables are defined. In line with Kogan et al. (2017): we omit financial firms (SIC codes 6000 to 6799) and utilities (SIC codes 4900 to 4949); we restrict attention to firm-year observations with nonnegative book assets and nonmissing SIC codes; we restrict attention to firms incorporated in the U.S.; and we winsorize at the 1% level using yearly breakpoints. Specifically, we winsorize innovation output, profit growth, and R&D-to-sales in the model and in the data. Compustat-related statistics are calculated for the same sample period as the productivity growth target, 1960–2019. Profit volatility is the standard deviation of profit growth, computed for all firms and for firms in the top profit quintile in the prior year.[43]

Following Bloom et al. (2013) and De Ridder (2024), R&D is the Compustat variable XRD. Innovation output is the sum of the economic value of all patents earned (based on stock market reaction to patent grants), normalized by firm value. The economic value of patents is obtained from the website of Dimitris Papanikolaou based on Kogan, Papanikolaou, Seru and Stoffman (2017).[44] As in their firm-level analysis, the data nor-

[43] Profit growth is measured as operating profits in a given year divided by operating profits in the prior year.

[44] See https://github.com/KPSS2017/Technological-Innovation-Resource-Allocation-and-Growth-Extended-Data.

malization uses the book value of firm assets AT. The innovation output distribution is similar when using firms' enterprise value in Compustat as a measure of firm value. Enterprise value is the sum of a firm's equity market capitalization, preferred stock outstanding, and the book value of debt. In the model, firm innovation output is the annual sum of the (scaled) economic value of each successful innovation, divided by (scaled) firm value $v^f(\sigma)$. This scaling is aligned with scaling by enterprise value in the data, as the model does not include long-term debt and hence the value of long-term debt in the model is zero. For leaders and tied firms ($\sigma \geq 0$), the scaled value of an innovation is $v^f(\sigma+1) - v^f(\sigma)$. For laggards, it is $v^f(0) - v^f(\sigma)$ after catch-up and $v^f(\sigma+1) - v^f(\sigma)$ after an incremental innovation. R&D-to-sales is annual R&D divided by annual sales. For R&D-to-sales moments in the estimation, we calculate, in the model and the data, the average ratio of R&D-to-sales, for all firms and for firms in the top quintile of firms ranked by profits. For the firm-level regressions of R&D on discount-rate gaps and hurdle rates, we merge Compustat with the firm-level discount-rate measures from Gormsen and Huber (2025). To reduce outliers, we restrict attention, in calculating profit growth between year y and $y+1$, in the model and the data, to firms with at least three quarters of positive operating profits in the base year y.

The top-left panel of Figure 8 reports estimates from regressions of the form:

$$\text{ProfitGrowth}_{f,t} = \varphi_{ind,t} + size'_{f,t-1}\vartheta + \varepsilon_{f,t},$$

where $\text{ProfitGrowth}_{f,t}$ is profit growth between t and $t+1$, $\varphi_{ind,t}$ is an industry-year fixed effect, $size_{f,t-1}$ is a vector of indicator variables for size quintile (with the largest quintile excluded and serving as the reference group), and ϑ is a coefficient vector. We cluster standard errors at the firm level. An industry is assigned to each firm using the two-digit level of the Standard Industrial Classification system.

Productivity. Total factor productivity growth is from Fernald et al. (2017). Foster et al. (2001) decompose productivity growth within industry j into five components. Let $\mathbb{C}_t$, $\mathbb{N}_t$, and $\mathbb{X}_t$ denote, respectively, continuing, entering, and exiting firms in industry j between $t-1$ and t. Let $\mathcal{I}_{jt}$ denote the set of firms active in industry j at time t. Firm productivity is log labor productivity, $\theta_{it} = \ln(y_{it}/l_{it})$. We denote firm i's revenue share in industry j by

$$\varsigma_{it} \equiv \frac{p_{it}y_{it}}{\sum_{h\in\mathcal{I}_{jt}} p_{ht}y_{ht}},$$

and aggregate productivity in industry j (suppressing the industry subscript) is

$$\Theta_t \equiv \sum_{i\in\mathcal{I}_{jt}} \varsigma_{it}\theta_{it}.$$

For continuing firms, define $\Delta\theta_{it} \equiv \theta_{it} - \theta_{it-1}$ and $\Delta\varsigma_{it} \equiv \varsigma_{it} - \varsigma_{it-1}$. The decomposition is

$$\Delta\Theta_t = \underbrace{\sum_{i\in\mathbb{C}_t} \varsigma_{it-1}\Delta\theta_{it}}_{\text{within}} + \underbrace{\sum_{i\in\mathbb{C}_t}(\theta_{it-1} - \Theta_{t-1})\Delta\varsigma_{it}}_{\text{between}} + \underbrace{\sum_{i\in\mathbb{C}_t}\Delta\theta_{it}\Delta\varsigma_{it}}_{\text{cross}} + \underbrace{\sum_{i\in\mathbb{N}_t}\varsigma_{it}(\theta_{it} - \Theta_{t-1})}_{\text{entry}} + \underbrace{\sum_{i\in\mathbb{X}_t}\varsigma_{it-1}(\Theta_{t-1} - \theta_{it-1})}_{\text{exit}}. \quad \text{(IA.55)}$$

When focusing on continuing firms, we calculate the within share as the within term divided by the sum of the within, between, and cross terms. In the model and the data, the FHK decomposition is calculated using observations five years apart.

Transition targets. For the ideas-hard-to-find exercise (Section 7), each data target is the change in a moment's decade average between the 1990s (1990–1999) and the 2010s (2010–2019). Productivity growth is from Fernald et al. (2017), R&D to GDP from BEA NIPA data, the mean net markup from Autor et al. (2020), and the discount-rate gap from Gormsen and Huber (2025), extended to 1990 using their mapping of Poterba and Summers (1995) as described in footnote 34.

E. Calibration and sensitivity

Table IA.1 shows the percent change in each targeted moment from a 1% increase in each indicated parameter or transformed parameter.

Firm growth dispersion, reallocation, and the labor share. Figure IA.4 shows that a rise in uninsured idiosyncratic risk, and the resulting endogenous discount-rate gap, implies lower firm growth dispersion, reduced job reallocation, and a fall in the labor share. Following Decker et al. (2016), we calculate firm growth dispersion as the cross-sectional standard deviation of employment growth and we calculate job reallocation as the sum of job creation and destruction. The labor share is ω (total wage bill as a share of output).

E.1 Wedge distribution

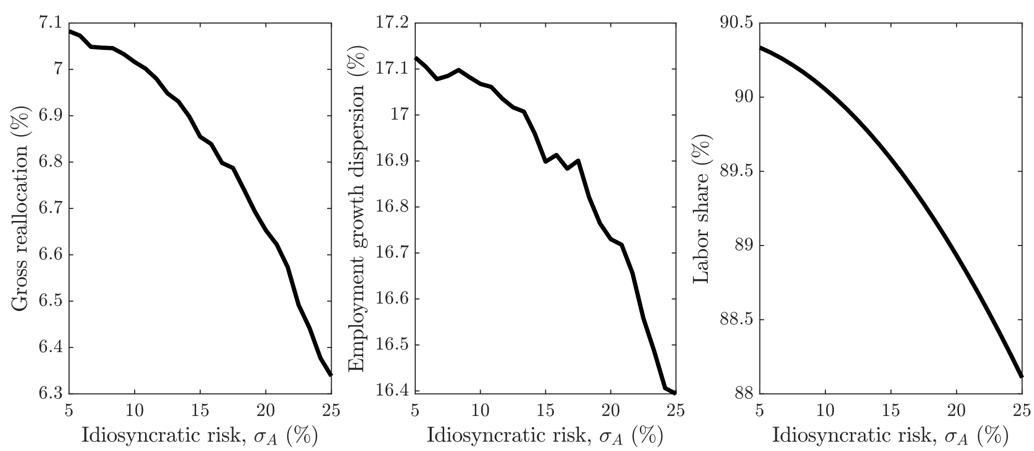
Figure 1 of the main text shows that the model matches key features of the full, non-targeted firm-level distributions of innovation output, markups, and the discount-rate gap κ. That figure reports balanced-growth-path distributions. When we instead use model-simulated panel data following a rise in κ due to higher idiosyncratic risk, the model better matches the left tail of the wedge distribution. In particular, in an example with aggregate κ rising from 1% to 4%, the 5th percentile is 1.00 percentage point in the model-simulated panel data, versus 0.79 percentage point in the data; the 10th percentile is 1.00 percentage point in the model, versus 1.17 percentage points in the data; and the 25th percentile is 1.54 percentage points in the model, versus 1.84 percentage points in

Table IA.1: **Sensitivity Matrix**
(percent change in each moment from a 1% increase in the indicated parameter or transformed parameter)

	ϕ	B	σ_A	$\lambda - 1$	$(1-\zeta)^{-1}$	α	ρ
Growth	-0.36%	1.20%	-0.04%	1.57%	0.09%	0.06%	-0.24%
Net markup	-1.47%	-0.39%	0.11%	1.17%	0.47%	-0.38%	0.71%
Cost of capital	-0.04%	0.12%	-0.55%	0.16%	0.01%	0.01%	1.15%
Discount-rate gap, κ	-0.58%	0.60%	1.58%	1.10%	0.94%	-1.19%	0.79%
Overall profit volatility	0.01%	1.65%	-0.25%	1.51%	1.25%	0.13%	-0.30%
Top profit volatility	0.81%	1.52%	-0.24%	0.88%	-0.21%	0.44%	-1.08%
Overall R&D-to-sales	-0.77%	0.60%	-0.13%	1.55%	0.39%	0.28%	-0.81%
Top R&D-to-sales	-0.34%	0.49%	-0.06%	1.05%	-1.11%	-0.28%	-1.70%
Mean IO	-0.06%	1.17%	-0.04%	0.74%	0.14%	0.29%	-0.31%
P90 IO	1.08%	2.93%	0.02%	-0.01%	-0.04%	-0.14%	-2.89%

This table reports the effect on targeted moments of a 1% increase in each calibrated parameter or transformed parameter. For the innovation step and product-market substitution, the relevant entries are expressed in terms of $\lambda - 1$ and $(1-\zeta)^{-1}$, respectively. IO is innovation output.

Figure IA.4: **Additional dynamism measures**



The figure plots firm growth dispersion, job reallocation, and the labor share as functions of uninsured idiosyncratic risk σ_A. Firm growth dispersion is the cross-sectional standard deviation of employment growth, and job reallocation is the sum of job creation and destruction (Decker et al. 2016). The labor share is ω (total wage bill as a share of output).

the data. Thus, the transition simulation brings the left tail of the wedge distribution substantially closer to the data.

E.2 Robustness

This section reports two robustness exercises. First, we vary externally calibrated parameters and then recalibrate the remaining parameters to match the same targeted moments as in the benchmark economy. We consider alternative values for the elasticity of intertemporal substitution, $\epsilon \in \{1.5, 2.5\}$, and for the R&D curvature parameter, $\gamma = 0.67$. Second, we introduce exogenous technology diffusion, and recalibrate the model parameters. Using these alternative calibrations, we explore the relation of uninsured idiosyncratic risk with the discount-rate gap, growth, markups, and firm valuations.

In the diffusion extension, exogenous technology diffusion closes the leader–laggard gap by one rung. It occurs at rate η. Thus, for $s > 0$, the law of motion for the gap distribution becomes

$$\begin{aligned}\dot{\mu}_t(s) = \; & \underbrace{\mu_t(s+1)\left[(1-\phi)x_t(-(s+1)) + \eta\right] + \mu_t(s-1)x_t(s-1)(1+\mathbb{1}_{s=1})}_{\text{Inflows to gap } s} \\ & - \underbrace{(x_t(s) + x_t(-s) + \eta)\mu_t(s)}_{\text{Outflows from gap } s},\end{aligned} \tag{IA.56}$$

with, as before, the convention $\mu_{\bar{s}+1}(t) = 0$. Relative to the benchmark law of motion, diffusion adds an inflow to gap s from gap $s+1$ and an outflow from gap s to gap $s-1$.

Table IA.2 reports the recalibrations. Across the exercises, the model continues to match the targeted moments reasonably well. The recalibrated economies continue to feature positive uninsured idiosyncratic risk, $\sigma_A > 0$, and limited pledgeability, $\alpha < 1$.

Figure IA.5 then varies uninsured idiosyncratic risk σ_A in each recalibrated economy. Across the exercises, a rise in σ_A raises the discount-rate gap κ, lowers productivity growth, raises markups, and increases valuation ratios. The markup increase is less pronounced when $\gamma = 0.67$ and especially when $\epsilon = 1.5$. Valuations rise most strongly for superstar firms.

E.3 The EIS and Hall-style regressions

We set the elasticity of intertemporal substitution to $\epsilon = 2$ in the benchmark calibration. This value is important for the comparative statics emphasized in the main text: with $\epsilon > 1$, a rise in uninsured idiosyncratic risk can raise firms' hurdle rates while lowering the financial cost of capital. The value of the EIS has been much debated in the literature. A standard empirical approach estimates the EIS from an aggregate Euler equation, relating consumption growth to the expected real risk-free rate. Based on this approach, Hall's (1988) results point to a very low EIS. We reproduce a Hall-style exercise in transition data from our model, using a shock to σ_A. We generate time series for entrepreneurial

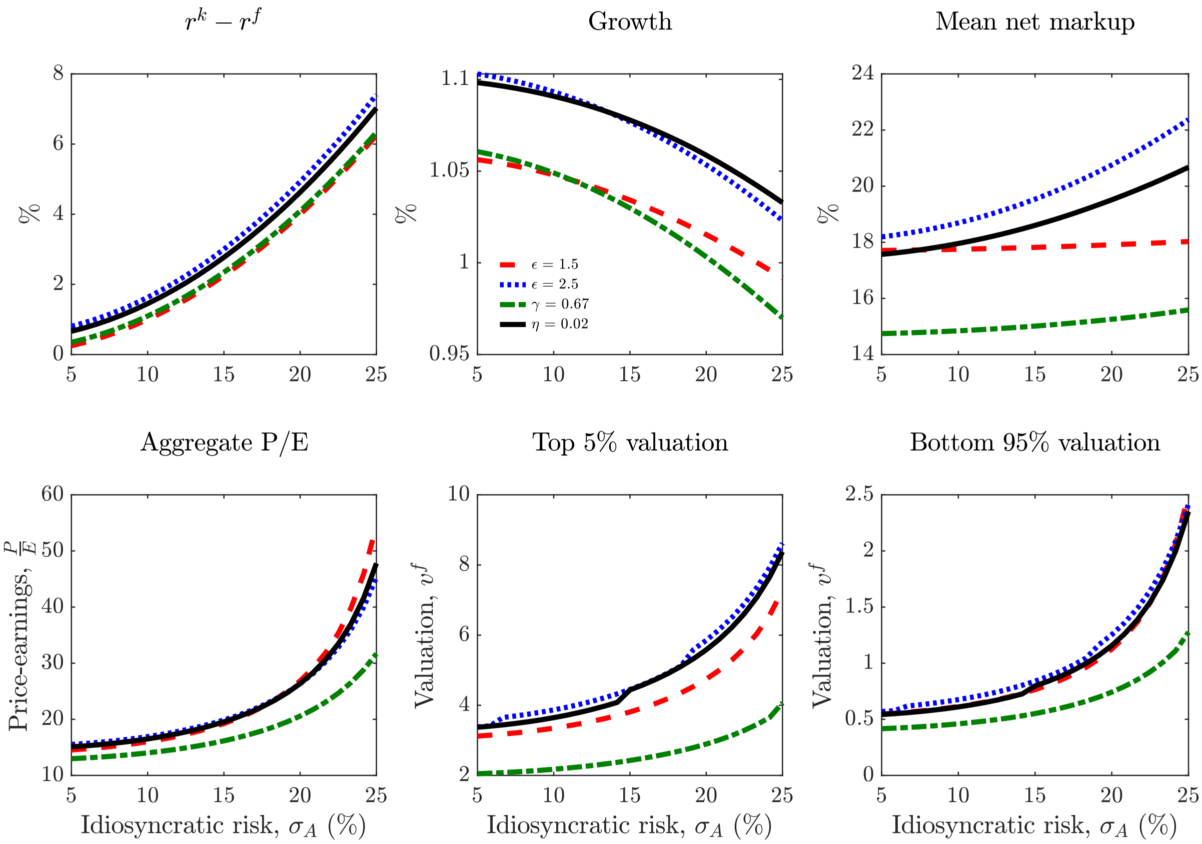
Table IA.2: **Robustness: Recalibrated parameters and targeted moments**

Moments	$\epsilon = 1.5$	$\epsilon = 2.5$	$\gamma = 0.67$	$\eta = 0.02$	Data
Productivity growth	1.03%	1.08%	1.03%	1.08%	1.03%
Mean net markup	17.84%	19.31%	15.06%	18.54%	19.41%
Cost of capital	4.92%	5.26%	5.95%	5.21%	5.20%
Discount-rate gap, κ	2.58%	2.64%	2.65%	2.64%	2.64%
Innovation output, mean	4.85%	4.49%	5.00%	4.52%	6.78%
Innovation output, 90th percentile	18.68%	19.04%	17.08%	19.11%	19.54%
Profit volatility, all firms	37.36%	35.72%	36.31%	35.81%	35.72%
Profit volatility, top quintile	16.14%	16.28%	17.57%	16.93%	19.48%
R&D to sales, all firms	4.42%	4.38%	4.50%	4.26%	4.37%
R&D to sales, top quintile	2.79%	3.32%	2.98%	3.55%	2.64%
Parameters					
ϕ	0.478	0.495	0.591	0.512	
$100 \cdot \ln(\lambda)$	3.941	4.395	3.597	4.506	
B	1.327	1.245	2.618	1.211	
α	29.67%	3.84%	14.23%	4.10%	
σ_A	16.07%	13.85%	15.99%	14.58%	
ρ	6.38%	6.18%	7.36%	6.27%	
ζ	0.977	0.975	0.973	0.976	
η	0.000	0.000	0.000	0.020	
γ	0.500	0.500	0.670	0.500	
ϵ	1.500	2.500	2.000	2.000	
SMM objective	0.502	0.487	0.749	0.536	

The table reports targeted moments and parameter values for robustness exercises. The first three columns vary an externally calibrated parameter and then recalibrate the remaining parameters. The fourth column introduces exogenous technology diffusion at rate $\eta = 0.02$ and recalibrates the remaining parameters.

consumption growth and the risk-free rate for an economy that begins on the benchmark balanced growth path with $\sigma_A = 14\%$. Later, at the start of the transition, there is a one-time, permanent, and unexpected increase in uninsured idiosyncratic risk to $\sigma_A = 21\%$. We regress entrepreneurial consumption growth on the risk-free rate, with both variables annualized and measured in percentage points (running this exercise in a BGP, without any shock, would produce constant consumption growth and a constant risk-free rate). The OLS estimate is 0.11 (s.e. 0.03), in line with the aggregate evidence (e.g., Hall 1988), even though the structural EIS in the simulated economy is $\epsilon = 2$. To see why this regression produces a downward-biased estimate of the EIS, note that, in our model, the Euler equation determining the risk-free rate includes the standard consumption growth term but also a precautionary-savings/risk-adjustment term. Movements in the risk-free rate induced by uninsured idiosyncratic risk are accompanied by shifts in this precautionary-saving/risk-adjustment term.

Figure IA.5: **Robustness: Varying uninsured idiosyncratic risk σ_A**



The figure plots outcomes as functions of uninsured idiosyncratic risk σ_A in the robustness exercises reported in Table IA.2. The panels show the discount-rate gap $\kappa = r^k - r^f$, productivity growth, the mean net markup, the aggregate price-earnings ratio, and firm valuations v^f for the top 5 percent and bottom 95 percent of firms.

Note also that cross-sectional evidence in Gruber (2013) using tax-induced variation in after-tax returns points to larger values: he estimates an EIS around 2 using variation in capital-income tax rates. Asset-pricing evidence is also often consistent with an EIS above one. In long-run-risk models, an EIS above one gives the empirically relevant comparative statics that higher expected long-run growth raises asset prices and higher uncertainty lowers them (Bansal and Yaron 2004). Estimating a long-run-risk model while accounting for time aggregation, Bansal et al. (2016) report an EIS estimate of 2.2. Our benchmark value of $\epsilon = 2$ is therefore consistent with tax-based and asset-pricing evidence. Moreover, our simulated Hall-style regression shows why low observed responsiveness of aggregate consumption growth to changes in the risk-free rate is not inconsistent with a high EIS in our environment.

F. Entry

We extend the benchmark economy to include firm entry. At each date, a mass one of potential entrants chooses an entry rate x_E and hires $G_E(x_E) = (x_E/B_E)^{1/\gamma}$ R&D workers, where B_E is entrants' R&D cost scalar. Conditional on a successful innovation, the entrant draws an industry with gap s with probability $\mu(s)$. In a tied industry, the entrant displaces one of the two incumbents at random and becomes the leader with a one-rung advantage. In an industry with gap $s > 0$, the entrant displaces the laggard; the innovation is incremental with probability $1 - \phi^E$, reducing the gap by one, and more-than-incremental with probability ϕ^E, placing the entrant ℓ^E rungs ahead of the old leader, with $\ell^E = 0$ corresponding to reaching the old leader's technology.

Let $v(\sigma)$ denote the scaled value of a firm in position σ. The expected value of a successful entry innovation is

$$\Delta v_E = \mu(0)v(1) + \sum_{s=1}^{\bar{s}} \mu(s) \left[(1 - \phi^E)v(1 - s) + \phi^E v(\ell^E)\right]. \tag{IA.57}$$

The entrant chooses x_E according to

$$G_E'(x_E)\omega = \Delta v_E. \tag{IA.58}$$

The model entry rate, measured per incumbent firm, is $x_E/2$.

Incumbent values reflect the risk of displacement and downgrading by entrants. Along a BGP, the scaled value of an incumbent in position σ satisfies

$$\left[\bar{\rho} - \left(1 - \tfrac{1}{\epsilon}\right)g\right] v(\sigma) = \pi(\sigma) - \omega G(x(\sigma)) + x(\sigma)\Delta v(\sigma) + x(-\sigma)\Delta^c v(\sigma) + x_E\, \Delta^E v(\sigma),$$

subject to the usual financial constraint, where the (negative) entry-induced capital gain is

$$\Delta^E v(\sigma) = \begin{cases} (1 - \phi^E)\, v(\sigma - 1) + \phi^E\, v(-\ell^E) - v(\sigma) & \text{if } \sigma \geq 1, \\ \tfrac{1}{2}\, v(-1) - v(0) & \text{if } \sigma = 0, \\ -v(\sigma) & \text{if } \sigma \leq -1. \end{cases} \tag{IA.59}$$

A laggard is displaced by any entry into its industry and loses its value; a tied incumbent is displaced with probability one half and otherwise becomes a one-rung laggard; and a leader keeps a one-rung-smaller lead after incremental entry or becomes an ℓ^E-rung laggard after more-than-incremental entry. We also add entrant R&D labor to labor-market clearing:

$$\sum_{s \in S^+} \mu(s) \left[G(x(s)) + G(x(-s)) + l(s) + l(-s)\right] + G_E(x_E) = 1. \tag{IA.60}$$

Entry also changes the law of motion for gaps. Let $\mathcal{I}_s(\mu, x)$ denote the incumbent-

Table IA.3: **Calibration with entry**

Parameter values			Model-data moments		
Parameter	Description	Value	Moment	Model	Data
ϕ	Laggard catch-up	0.534	Productivity growth	1.25%	1.03%
ϕ^E	Entrant catch-up probability	0.086	Mean net markup	19.34%	19.41%
$\ln(\lambda)$	Innovation step size	3.49%	Cost of capital, r^f	6.26%	5.20%
ζ	CES parameter	0.987	Discount-rate gap, κ	2.65%	2.64%
B	Incumbent R&D cost scalar	1.553	Entry rate	8.24%	11.70%
B_E	Entrant R&D cost scalar	1.736	Employment share, age < 2	4.94%	4.96%
α	Pledgeability	6.75%	Employment share, age < 4	13.95%	9.48%
σ_A	Idiosyncratic risk	3.27%	Innovation output, mean	6.75%	6.78%
ρ	Time preference	5.72%	Innovation output, 90$^{\text{th}}$ percentile	19.56%	19.54%
ℓ^E	Entrant rung	0	Profit volatility, all firms	36.58%	35.75%
			Profit volatility, top quintile	17.64%	19.45%
			R&D to sales, all firms	3.06%	4.37%
			R&D to sales, top quintile	4.82%	2.65%

The table reports calibrated parameters and selected model-data moments for the model with entry. Rates and rate-like parameters are reported using the same annualized conventions as in Table 1; profit volatility is computed from annual firm-level profit growth and is not annualized. The parameter ℓ^E is the entrant's step relative to the old leader after a more-than-incremental entry innovation; $\ell^E = 0$ means the entrant reaches the old leader's technology.

innovation terms in Eq. (4). For $s = 1, \ldots, \bar{s}$,

$$\dot{\mu}(s) = \mathcal{I}_s(\mu, x) + x_E \left[(1-\phi^E)\mu(s+1) + \mathbb{1}_{s=1}\,\mu(0) + \phi^E\,\mathbb{1}_{s=\ell^E} \sum_{m=1}^{\bar{s}} \mu(m) - \mu(s) \right], \quad \text{(IA.61)}$$

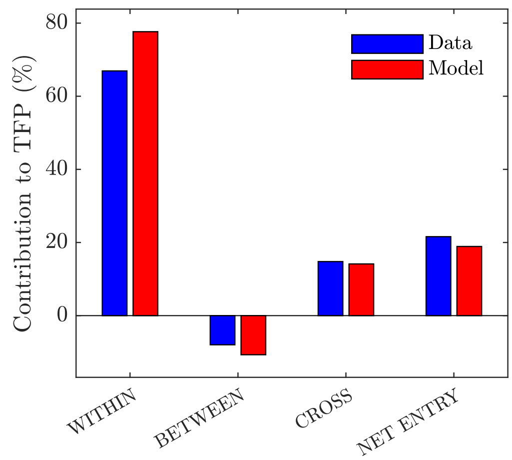
with $\mu(\bar{s}+1) = 0$ and $\mu(0)$ determined by $\sum_{s=0}^{\bar{s}} \mu(s) = 1$. When $\ell^E = 0$, more-than-incremental entry into an industry with a positive gap moves the industry to the tied state; this inflow is captured through the normalization determining $\mu(0)$. Aggregate productivity growth is

$$g = \ln\lambda \left[\sum_{s \in S^+} \mu(s)(1+\mathbb{1}_{s=0})x(s) + x_E \left(\mu(0) + \phi^E \ell^E \sum_{s=1}^{\bar{s}} \mu(s) \right) \right]. \quad \text{(IA.62)}$$

We re-estimate the model after adding the entry parameters (B_E, ϕ^E, ℓ^E), targeting the entry rate, young-firm employment shares, and the net-entry component of the Foster et al. (2001) decomposition, while retaining the aggregate and cross-sectional moments from the benchmark calibration.

Table IA.3 reports the calibrated parameters and selected model-data moments. Overall, the estimated model fits the moments, including the new entry moments, reasonably well. Figure IA.6 compares the model and data for the Foster et al. (2001) decomposition. The model captures the broad role of incumbent reallocation and the net-entry margin.

Figure IA.6: **FHK decomposition with entry**



The figure compares the model and data for the Foster et al. (2001) productivity-growth decomposition. Each column shows the contribution to productivity growth divided by productivity growth.

Note that while the calibrated value of σ_A is lower than in the baseline model without entry, the comparative statics with respect to σ_A have similar key features. Figure IA.7 shows how discount rates, growth, markups, entry, and valuations vary with uninsured idiosyncratic risk σ_A. A rise in σ_A widens the gap between the hurdle rate and the financial cost of capital: r^k rises, while r^f falls. The higher hurdle rate weakens incumbent innovation and discourages entry, so productivity growth and the entry rate decline. (Growth falls more sharply than in the benchmark model without entry.) At the same time, weaker creative destruction makes market leadership more durable, raising the mean net markup, firm valuations, and the aggregate price-earnings ratio. The valuation gains are concentrated among superstar firms.

G. Transition dynamics and algorithm

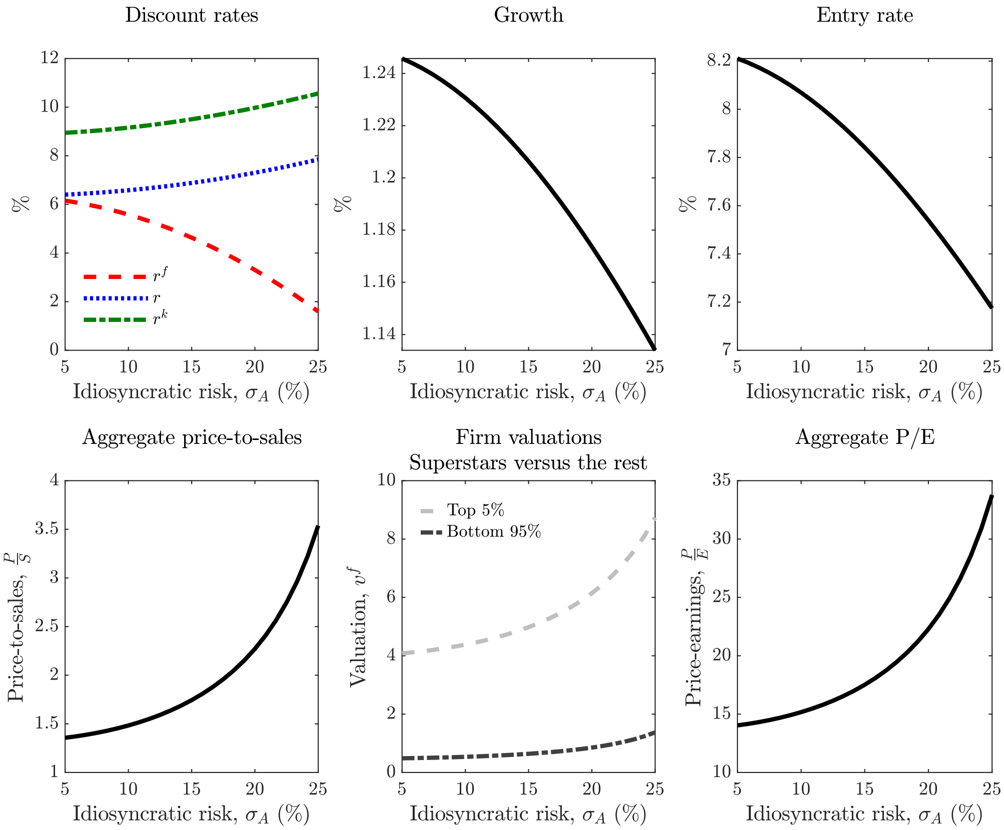
G.1 Rise in idiosyncratic risk

Figure IA.8 shows the transition dynamics following a one-time, unexpected, and permanent rise in idiosyncratic risk from $\sigma_A = 14\%$ to $\sigma_A = 21\%$, holding all other parameters fixed. The figure plots responses relative to the initial balanced growth path for discount rates, productivity growth, markups, and valuation ratios.

G.2 Transition algorithm

We compute the transition path between an initial BGP ($t = 0$) and a final BGP using an iterative shooting algorithm with alternating updates. The transition algorithm uses firm values scaled by entrepreneurs' consumption, $v_t^C(\sigma) \equiv V_t(\sigma)/C_t^E$, rather than firm values

Figure IA.7: **Entry model: Varying uninsured idiosyncratic risk σ_A**

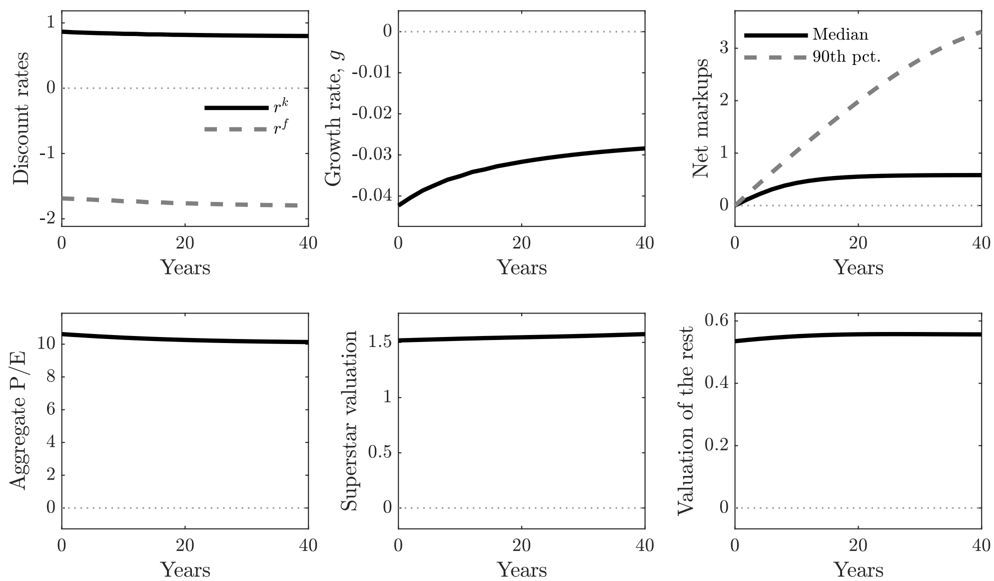


The figure plots outcomes as functions of idiosyncratic risk σ_A in the model with entry. The panels show discount rates (r^f, r, and r^k), productivity growth, the entry rate, the mean net markup, firm valuations v^f for the top 5 percent and bottom 95 percent of firms, and the aggregate price-earnings ratio.

scaled by output, $v_t(\sigma) \equiv V_t(\sigma)/Y_t$. Given the path for v^C and ω, one immediately obtains the path for v as $v_t(\sigma) = (1-\omega_t)v_t^C(\sigma)$. The algorithm works with the continuous-time HJB representation for firm value V.

Time grid. We discretize the transition horizon into T periods using a non-uniform time grid. The first segment uses a finer step size Δt_1 (covering the first 100 years) and the second uses a coarser step size Δt_2 for the remainder. This concentrates computational effort during the "early" transition, where dynamics change more rapidly; if convergence to the new BGP is not achieved in the early transition, the economy completes the convergence process later with a coarser step size. Parameters that shift between steady states (e.g., ϕ, B, γ, σ_A) may either jump at date 0, as in the exercises reported in the main text, or follow prescribed paths from their initial to their final steady-state values over a transition window $[0, T_{\text{end}}]$, after which they remain at their terminal values. In the backward value update, $g^E_{t+\Delta t}$ denotes the guessed entrepreneurial-consumption growth

Figure IA.8: **Transition dynamics after a rise in idiosyncratic risk σ_A**



The figure shows transition dynamics following a one-time, unexpected, and permanent rise in idiosyncratic risk from $\sigma_A = 14\%$ to $\sigma_A = 21\%$, holding all other parameters fixed. Responses are plotted relative to the initial balanced growth path (i.e., subtracting the value from the initial BGP). The panels show discount rates r^k and r^f, productivity growth, the median and 90th percentile markup, aggregate P/E, superstar valuation, and valuation of the remaining firms.

rate over the interval from t to $t + \Delta t$.

Initialization. We initialize guesses for the wage share path $\{\omega_t\}_{t=0}^{T}$ and the entrepreneurial consumption growth rate path $\{g^E_{t+\Delta t}\}_{t=0}^{T}$ by linearly interpolating between the initial and final steady-state values over $[0, T_{\text{end}}]$, and holding them constant at the final steady-state values thereafter. Note that along a balanced growth path $g^E_{t+\Delta t}$ coincides with the aggregate growth rate. Note also that $\{g^E_{t+\Delta t}\}_{t=0}^{T}$ is always stored as an annualized growth rate.

Algorithm. The algorithm has two nested loops.

The outer loop alternates between updating the wage share path (on odd iterations) and the growth rate path (on even iterations). This continues until the maximum error across both paths falls below a convergence criterion.

Given current guesses $\{\omega_t\}_{t=0}^{T}$ and $\{g^E_{t+\Delta t}\}_{t=0}^{T}$, each inner iteration proceeds in four steps.

Step 1: Backward shooting (value functions and investment). Starting from the terminal steady-state value functions and innovation rates, we solve backward for the value functions and investment policies. The discretization uses a backward-Euler step, so continuation values in the innovation gains are evaluated at $t + 1$. Let $\text{Prob}(s' \mid s)$ and

$\mathrm{Prob}^c(s' \mid s)$ denote the transition kernels associated with own and competitor innovation, respectively. At each t, we first compute

$$\Delta v_t^C(s) = \sum_{s'} \mathrm{Prob}(s' \mid s)\, v_{t+1}^C(s') - v_{t+1}^C(s), \tag{IA.63}$$

$$\Delta^c v_t^C(s) = \sum_{s'} \mathrm{Prob}^c(s' \mid s)\, v_{t+1}^C(s') - v_{t+1}^C(s), \tag{IA.64}$$

where $\Delta v_t^C(s)$ is the expected capital gain from own innovation. The object $\Delta^c v_t^C(s)$ is the expected capital gain associated with competitor (c) innovation; it is weakly negative, so equivalently $-\Delta^c v_t^C(s)$ is the weakly positive expected capital loss from competitor innovation. Investment is then given by the first-order condition

$$x_t^*(s) = (G')^{-1}\left(\max\{\Delta v_t^C(s),\, 0\} \cdot \frac{1-\omega_t}{\omega_t}\right), \tag{IA.65}$$

if the financing constraint does not bind. Thus, realized innovation is

$$x_t(s) = \min\left\{x_t^*(s), G^{-1}\left(\alpha\, v_t^C(s) \Big/ \left(\frac{\omega_t}{1-\omega_t}\right)\right)\right\}.$$

Let

$$\bar{\rho}_t \equiv \rho + \frac{1}{2}\left(1 - \frac{1}{\epsilon}\right)\nu\sigma_{A,t}^2,$$

which is the time-preference rate in the no-idiosyncratic-risk Schumpeterian block that gives the same entrepreneurial required return. The value function satisfies the discretized HJB equation

$$v_t^C(s) = \left(\bar{\rho}_t + \frac{1-\epsilon}{\epsilon}\, g_{t+\Delta t}^E + \frac{1}{\Delta t}\right)^{-1}\left[\frac{\pi(s)}{1-\omega_t} - \frac{\omega_t}{1-\omega_t}G(x_t(s)) + \Delta v_t^C(s)\, x_t(s) \right.$$
$$\left. + \Delta^c v_t^C(s)\, x_t^c(s) + \frac{v_{t+1}^C(s)}{\Delta t}\right], \tag{IA.66}$$

where $x_t^c(s)$ denotes the competitor's investment rate. Other current-period variables are indexed by t.

Step 2: Forward shooting (firm distributions). Starting from the initial distribution μ_0, we propagate the firm distribution forward. At each t, we construct the Markov transition matrix M_t from the full set of innovation rates $\{x_t(\sigma)\}_{\sigma\in S}$. The distribution updates as $\mu_{t+1} = \mu_t\, M_t$.

Step 3: Labor market clearing. At each t, we solve for the wage share $\hat{\omega}_t$ that clears the labor market. The labor market clearing condition is given by Eq. (IA.39); note that production labor is decreasing in the wage share as discussed in Internet Appendix C.1.

Step 4: Entrepreneurial consumption growth rate. We compute the implied growth rate of entrepreneurial consumption over each interval $[t, t+\Delta t]$. Let m_s denote the leader's relative price in a gap-s industry, and let $\psi(s)$ and $\psi(-s)$ denote the leader and follower gross markups in Eq. (IA.9). Define the industry price index relative to the leader's marginal cost,

$$\Psi_s \equiv \left[\psi(s)^{-\frac{\zeta}{1-\zeta}} + (\lambda^s\psi(-s))^{-\frac{\zeta}{1-\zeta}}\right]^{-\frac{1-\zeta}{\zeta}}, \tag{IA.67}$$

so that $\ln \Psi_s = -\Phi(s)$, with $\Phi(s)$ defined in Eq. (IA.43).

The implied growth rate is

$$g^E_{t+\Delta t} = \underbrace{\ln(\lambda)\sum_{s\in S^+}\mu_t(s)x_t(s)\big(1+\mathbb{1}_{s=0}\big)}_{\text{innovation along the frontier}} - \underbrace{\sum_{s\in S^+}\dot{\mu}_t(s)\ln\Psi_s}_{\text{composition effect}} - \underbrace{\frac{\dot{\omega}_t}{\omega_t(1-\omega_t)}}_{\text{wage share effect}}. \tag{IA.68}$$

where $\dot{\mu}_t(s) = (\mu_{t+1}(s) - \mu_t(s))/\Delta t$ and $\dot{\omega}_t = (\omega_{t+1} - \omega_t)/\Delta t$. The first term captures productivity growth from innovation steps, where tied industries ($s = 0$) receive weight 2 because both firms operate at the frontier; the second reflects how a changing gap distribution moves output given the frontier; and the third captures how a changing labor share moves both output and entrepreneurs' share of it. Equation (IA.68) is the discrete-time counterpart of combining the output-growth condition (IA.44) with $g^E_t = g^Y_t - \dot{\omega}_t/(1-\omega_t)$, which follows from $C^E_t = (1-\omega_t)Y_t$. It reduces to Eq. (15) on a BGP, where $\dot{\mu}_t(s) = \dot{\omega}_t = 0$.

Updating and convergence. After each inner loop, we compare the implied path $\hat{z}_t$ to the guessed path z_t (where $z \in \{\omega, g^E\}$ depending on the outer iteration) and apply damped updating:

$$z^{\text{new}}_t = (1-\lambda_{\text{upd}})\,\hat{z}_t + \lambda_{\text{upd}}\,z^{\text{old}}_t, \tag{IA.69}$$

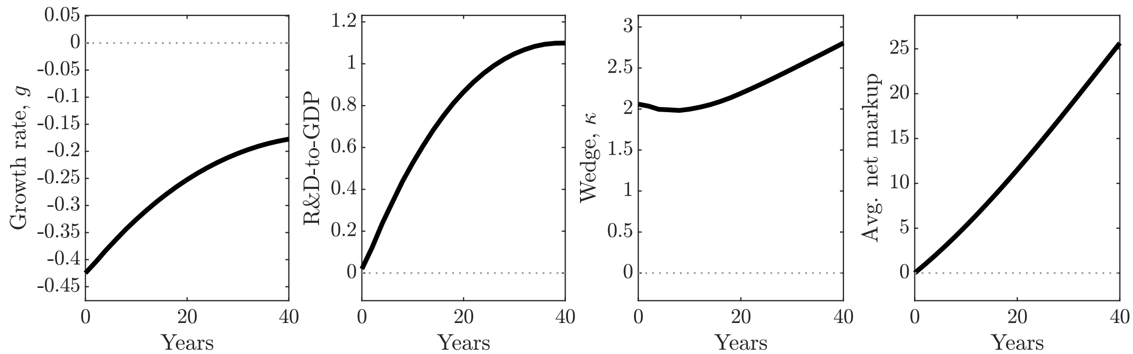
where $\lambda_{\text{upd}} \in (0,1)$ is a damping parameter. The inner loop iterates until the maximum relative error $\max_t |(\hat{z}_t - z_t)/z_t|$ falls below a tolerance. The outer loop terminates when the errors for the wage share and growth rate are each below a tolerance.

G.3 Transition dynamics for ideas-hard-to-find exercise

Figure IA.9 plots the transition path for the calibrated change in $(\phi, \gamma, B, \sigma_A)$ reported in Table 4. The year-20 values of the plotted paths correspond to the model moments reported in Table 4. Productivity growth falls notably on impact and then partly recovers. Output growth falls more than productivity growth because wider technology gaps and higher markups reduce output conditional on a given technology frontier. The discount-rate gap jumps on impact (reflecting the rise in σ_A) and then increases further over time, as technology gaps widen. R&D to GDP and the markup rise more gradually as the economy becomes less competitive.

H. Additional nontargeted moments

Figure IA.9: **Transition dynamics: Higher idiosyncratic risk and ideas harder to find**



The figure shows transition dynamics following the permanent change in $(\phi, \gamma, B, \sigma_A)$ reported in Table 4. Responses are plotted relative to the initial balanced growth path.

H.1 Additional discount-rate gap and transition-rate moments

This subsection reports additional nontargeted moments related to the firm-level patterns discussed in Section 5. Table IA.4 presents panel regressions of firm-level discount-rate gaps (and, in the data, hurdle rates) on initial market power interacted with a linear year trend, using model-simulated panels along the balanced growth path and following the σ_A and α shocks. Table IA.5 reports model-simulated regressions of firm-level discount-rate gaps on initial idiosyncratic risk and initial financial constraints, with financial constraints measured using the Hadlock and Pierce (2010) index applied to the simulated data.

Figure IA.10 complements the firm-dynamics analysis by reporting transition rates across firm-size and discount-rate-wedge categories. The transition rates are the shares of observations moving from one category to another; firm size is measured by profits, and all moments are reported in percent.

H.2 Innovation output and future firm growth

Kogan et al. (2017) present evidence that a firm's own innovation output is associated with a persistent rise in its profits and revenue productivity, while the innovation output of a firm's competitor is associated with declines in profits and revenue-based productivity (TFPR). Following their approach, using model-simulated data aggregated to an annual frequency, we estimate regressions of the form:

$$\log\left(\frac{X_{f,t+5}}{X_{f,t}}\right) = a\,\mathrm{IO}_{f,t} + b\,\mathrm{IO}_{f^c,t} + cZ_{f,t} + u_{f,t}, \tag{IA.70}$$

where $X_{f,t}$ is, in turn, TFPR and profits for firm f; $\mathrm{IO}_{f,t}$ is firm f's innovative output (as described in Internet Appendix D); $\mathrm{IO}_{f^c,t}$ is the innovative output of firm f's competitor;

Table IA.4: **Market power and the secular evolution of discount rates and wedges**

	Model			Data	
	Along BGP	Following σ_A shock	Following α shock	Gap	Hurdle
Initial market power × year	0.010	0.016	0.058	0.083	0.094
	(0.001)	(0.002)	(0.005)	(0.046)	(0.050)
Fixed effects	Firm	Firm	Firm	Firm	Firm

This table reports panel regressions of firm-level discount-rate gaps or hurdle rates on initial market power interacted with a linear year trend. The first row reports the coefficient on this interaction. In the model columns, the dependent variable is the discount-rate gap; the columns use a balanced-growth-path simulation, a simulation following the σ_A shock, and a simulation following the α shock. Initial market power is the firm's pre-shock net markup, standardized to unit standard deviation. The two data columns report the corresponding discount-rate-gap and hurdle-rate coefficients from Gormsen and Huber (2025), where initial market power is averaged over 2000–2002 and the number of observations is 976. All regressions include the year trend and firm fixed effects. Each model column has 46,200 observations. Standard errors, in parentheses, are clustered by firm in the model columns and by firm and quarter in the data columns.

Table IA.5: **Differences in wedges across firms**

Specification	Along BGP	Following σ_A shock	Following α shock
Risk	0.152	0.593	1.274
	(0.011)	(0.030)	(0.054)
Financial constraints	0.240	0.921	1.847
	(0.008)	(0.025)	(0.049)
Fixed effects	Time	Time	Time

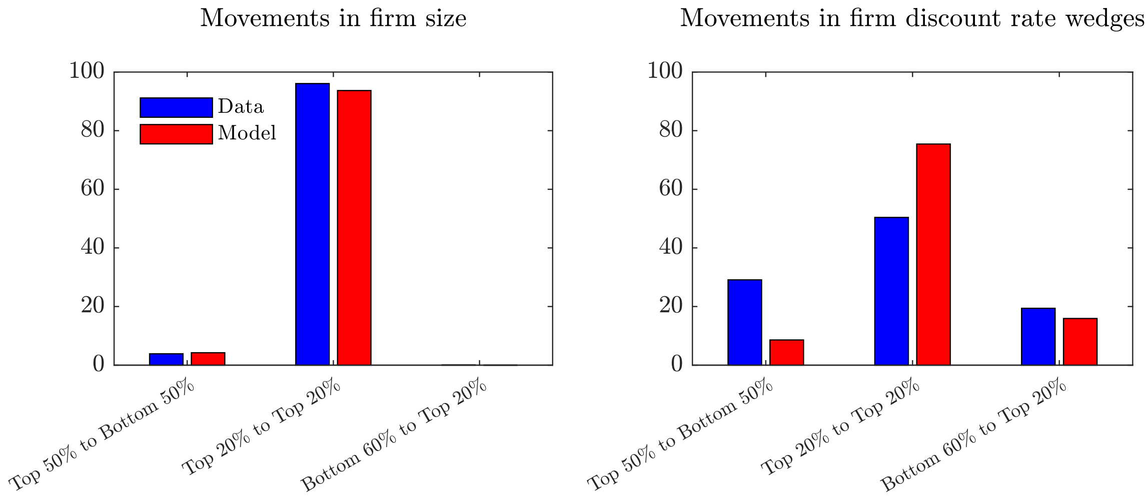
This table reports model-simulated panel regressions of the firm- level discount-rate gap on idiosyncratic risk, measured as the standard deviation of one-year idiosyncratic returns, and financial constraints, measured using minus initial sales as the model analogue of Hadlock and Pierce (2010). Each row block reports a separate univariate regression. Risk and financial constraints are measured at the start of the simulated panel and standardized to unit standard deviation. The first column uses a balanced-growth-path simulation; the second uses a simulation following the σ_A shock; and the third uses a simulation following the α shock. All regressions include time fixed effects. Standard errors, in parentheses, are clustered by firm. Coefficients are in percentage points. Columns in the "Risk" row have 44,856; 44,982; and 44,982 observations, respectively. All columns in the "Financial constraints" row have 46,200 observations.

and the vector Z includes time fixed effects, log firm value, and log firm employment.[45] We cluster standard errors by both firm and year. Independent variables are normalized to unit standard deviation.

Table IA.6 reports the regression results. We focus on the estimates of a and b, which

[45]As in Kogan et al. (2017) and our model estimates reported in Table IA.6, profits are gross profits, or sales minus cost of goods sold.

Figure IA.10: **Transition rates: Movements in firm size and firm discount-rate gaps**



Transition rates are the share of observations moving from one size or wedge category to another. Firm size is measured by profits. All moments are shown in percent.

capture the impacts of a firm's own innovation and competitor innovation, respectively. Data estimates are from Kogan et al. (2017). In the data, over a five-year horizon, a one standard deviation increase in a firm's innovation is associated with a 2.4% increase in TFPR and a 4.6% increase in profits. A one standard deviation increase in innovation by a firm's competitor is associated with a decline of 1.7% in TFPR and a 3.8% decline in profits. The estimated model broadly matches the pattern in the data: own innovation output is associated with higher profits and revenue productivity, while competitor innovation output is associated with lower profits and revenue productivity. The coefficient estimates in the simulated data are highly statistically significant and their magnitudes are smaller than those in the data for TFPR and larger for firm profits.

H.3 Heterogeneous pledgeability

Our benchmark model features rich endogenous firm heterogeneity. Firms differ in their initial labor productivity and technology position σ, and the subsequent evolution of each firm's technology position depends on its own innovation strategy, its competitor's innovation strategy, and stochastic innovation arrivals. This endogenous heterogeneity is sufficient for our main analysis: it generates strategic interactions that shape the aggregate response to changes in idiosyncratic risk and pledgeability, and it produces cross-sectional variation (consistent with the data) in discount-rate gaps, innovation output, and markups. For example, the model matches the distributions of innovation output, markups, and κ reported in Figure 1. It also matches cross-sectional patterns related to firms' initial technology gaps and market power: firms with greater initial

Table IA.6: **Innovation output and future firm growth**

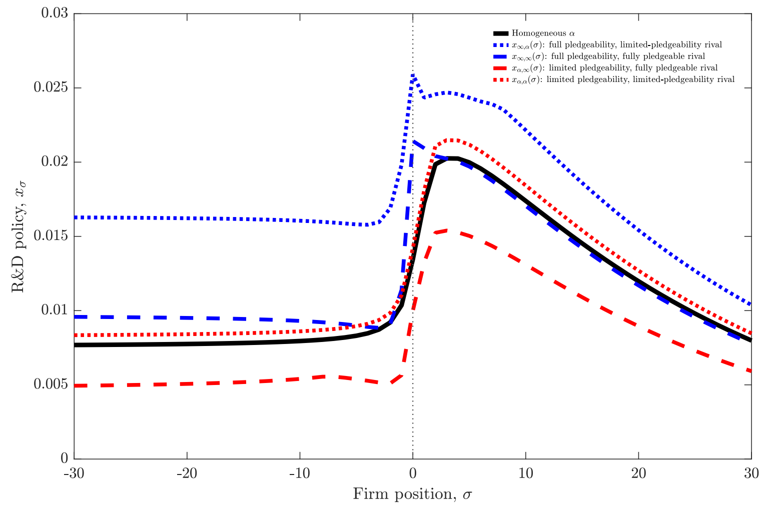
	Own innovation		Competitor innovation	
	TFPR	Profits	TFPR	Profits
A. Model	0.629	9.454	-1.041	-12.796
	(0.017)	(0.207)	(0.031)	(0.238)
B. Data	2.400	4.600	-1.700	-3.800
	(0.557)	(1.296)	(0.391)	(0.650)

This table reports estimates of Eq. (IA.70), following the empirical specification in Kogan et al. (2017). The dependent variable is five-year growth in TFPR or profits. Own innovation and competitor innovation are standardized to unit variance. The controls are time fixed effects, log firm value, and log firm employment. Standard errors, in parentheses, are clustered by firm and year. All coefficients are shown in percentage points. Model columns have 118,000 observations.

market power experience larger subsequent increases in wedges and discount rates, as shown in Figure 7 and Table IA.4. In addition, Gormsen and Huber (2025) show that firms facing greater financial constraints according to the index of Hadlock and Pierce (2010) maintain higher discount rates and wedges, as do firms with higher idiosyncratic return volatility; Internet Appendix Table IA.5 shows that these results obtain in model-simulated data as well.

An additional theoretical margin is ex-ante type heterogeneity. In the benchmark model, all firms have the same pledgeability parameter α. In this stylized exercise, we explore heterogeneity in pledgeability, illustrating how ex-ante pledgeability differences shape innovation incentives and leadership rather than recalibrating the benchmark economy. For tractability, we assume that each industry has one firm that can pledge the full value of its innovation profits and one firm that can pledge only a share α. Innovation strategies then depend on both the firm's own pledgeability and its competitor's pledgeability. We denote by $x_{\infty,\alpha}(\sigma)$ the innovation strategy of a fully pledgeable firm at technology position σ when its competitor has limited pledgeability. Full pledgeability corresponds to $\alpha = 1$: a firm's flow of R&D spending is far below its value at the optimal policies, so the constraint $\omega G(x) \leq v$ never binds. The subscript ∞ therefore denotes the absence of a binding pledgeability constraint (rather than a value of the pledgeability share). We denote by $x_{\alpha,\infty}(\sigma)$ the innovation strategy of a firm with limited pledgeability when its competitor is fully pledgeable. We also characterize two additional strategies: $x_{\infty,\infty}(\sigma)$, the strategy of a fully pledgeable firm facing a fully pledgeable competitor, and $x_{\alpha,\alpha}(\sigma)$, the strategy of a limited-pledgeability firm facing a limited-pledgeability competitor. Because each industry has exactly one fully pledgeable firm and one limited-pledgeability firm, the configurations underlying $x_{\infty,\infty}$ and $x_{\alpha,\alpha}$ arise for zero mass of

Figure IA.11: **Innovation policy with heterogeneous pledgeability**



The figure plots firms' equilibrium innovation policies, conditional on technology position σ, with heterogeneous pledgeability. The solid black line is the homogeneous-pledgeability benchmark. Blue lines correspond to fully pledgeable firms and red lines correspond to firms with limited pledgeability. Dashed lines indicate that the firm's competitor is fully pledgeable; dotted lines indicate that the firm's competitor has limited pledgeability.

industries. These equilibrium innovation rates can still be calculated, and would apply in industries with the state configurations (i.e., both firms have limited pledgeability, or neither firm does) with those industries having infinitesimal mass. Thus, these strategies are still useful for isolating how firms' own pledgeability and their competitors' pledgeability shape strategic innovation incentives.

Figure IA.11 plots the equilibrium innovation strategies. Fully pledgeable firms innovate more than limited-pledgeability firms at the same technology position, holding fixed the competitor's type. This comparison is shown by the blue lines lying above the corresponding red lines. Competitor type also matters: facing a fully pledgeable competitor lowers a firm's innovation rate, because a more innovative competitor raises the probability that the benefits of the firm's own innovation are eroded by future creative destruction. This force is shown by the dashed lines lying below the corresponding dotted lines. The solid black line reports the homogeneous-pledgeability benchmark.

Firms that can fully pledge innovation profits and face a competitor with limited pledgeability innovate at the highest rate for any given technology position: $x_{\infty,\alpha}(\sigma)$ lies above the other policies, while $x_{\alpha,\infty}(\sigma)$ lies below them. The gap between the two reflects both the direct relaxation of the firm's own constraint and the strategic discouragement effect of facing a better-financed rival.

This type heterogeneity also affects the distribution of market leadership. Fully pledgeable firms tend to hold stronger technology positions than firms with limited pledgeability. In the stationary distribution, the median technology position is 5 for fully pledgeable firms and -5 for firms with limited pledgeability. Fully pledgeable firms are leaders in 83% of industries, compared with 7% for firms with limited pledgeability; tied industries account for the remaining share. Thus, when pledgeability differs across firms, the less financially constrained firms are more likely to become market leaders, as expected.

I. Alternative mechanisms

The pricing-power and entry-barrier experiments in Section 6 require no new model elements: the former raises ζ in the calibrated benchmark economy and the latter lowers B^E in the calibrated entry economy of Internet Appendix F, in each case holding all other parameters at their calibrated values. This appendix details the third experiment, which modifies the asset-pricing block.

I.1 Convenience benefits of government debt and a higher equity premium

This appendix describes the higher-equity-premium analysis in Section 6. The analysis introduces a wedge between the financial cost of capital and the yield on government debt. This wedge captures the idea that government debt provides safety or liquidity services that are not provided by claims on private firms. We represent this force as a return-equivalent convenience-yield wedge.

Let $\varrho \geq 0$ denote the wedge between the financial cost of capital on diversified private claims and the yield on government debt. Let r^f_ϱ be the financial cost of capital and r^{gov}_ϱ the government-debt yield. The convenience wedge implies

$$r^f_\varrho - r^{\text{gov}}_\varrho = \varrho. \tag{IA.71}$$

The non-convenience claim and government debt are in zero net supply, so their introduction does not change aggregate resource feasibility or the model's creative-destruction mechanics and laws of motion (equilibrium innovation rates will change, however). As in the benchmark model, these zero-net-supply safe claims are priced using entrepreneurs' marginal valuation of infinitesimal payoffs at the equilibrium allocation.

To keep the exercise centered around the benchmark financial-cost-of-capital expression, define

$$\bar{r}(g_\varrho) \equiv \rho + \frac{g_\varrho}{\epsilon} - \frac{1}{2}\left(1 + \frac{1}{\epsilon}\right)\nu\sigma_A^2. \tag{IA.72}$$

We implement the convenience-yield experiment as

$$r_\varrho^f = \bar{r}(g_\varrho) + \frac{1}{2}\varrho, \tag{IA.73}$$

$$r_\varrho^{\text{gov}} = \bar{r}(g_\varrho) - \frac{1}{2}\varrho. \tag{IA.74}$$

The split in Eqs. (IA.73)–(IA.74) gives a spread of ϱ between private and government safe claims while preserving the benchmark expression as the midpoint.

The return required by entrepreneurs to own and operate innovative firms continues to include the idiosyncratic-risk premium. Along the same balanced growth path,

$$r_\varrho = r_\varrho^f + \nu\sigma_A^2 = \rho + \frac{g_\varrho}{\epsilon} + \frac{1}{2}\left(1 - \frac{1}{\epsilon}\right)\nu\sigma_A^2 + \frac{1}{2}\varrho. \tag{IA.75}$$

Thus, the primitive convenience-yield wedge does not directly affect the discount-rate gap studied in our paper. The firm-level gap remains

$$\kappa_\varrho(\sigma) = r_\varrho^k(\sigma) - r_\varrho^f, \tag{IA.76}$$

where $r_\varrho^k(\sigma)$ is the hurdle rate implied by the firm's R&D problem. A spread measured relative to the government-debt yield would instead equal

$$r_\varrho^k(\sigma) - r_\varrho^{\text{gov}} = \kappa_\varrho(\sigma) + \varrho. \tag{IA.77}$$

That spread includes the convenience yield on government debt and is therefore distinct from the discount-rate gap κ.

For computation, the higher-equity-premium experiment changes the Schumpeterian block through the entrepreneurial required return in Eq. (IA.75). The balanced-growth value equation is the counterpart of the benchmark BGP equation for scaled firm value:

$$(r_\varrho - g_\varrho)v_\varrho(\sigma) = \pi(\sigma) - G(x(\sigma))\omega + x(\sigma)\Delta v_\varrho(\sigma) + x(-\sigma)\Delta^c v_\varrho(\sigma), \tag{IA.78}$$

subject to the same financing constraint $\omega G(x(\sigma)) \leq \alpha v_\varrho(\sigma)$ and innovation technology as in the main model. The government-debt yield in Eq. (IA.74) is then recovered from the equilibrium financial cost of capital using Eq. (IA.71).

J. Present-value decomposition of firm valuations

This appendix applies the within-firm present-value decomposition of Cho, Grotteria, Kremens and Kung (2026) to firm-level data simulated along the transition studied in Section 7. We begin with the log-linear present-value identity in Cho et al. (2026). Let $m_{i,t} \equiv \log(M_{i,t}/Y_{i,t})$ denote firm i's log market value-to-output ratio, where $M_{i,t}$ is firm value and $Y_{i,t}$ is output. In this subsection, we use their notation; $(k, \rho, \phi_1, \phi_2)$ are log-linearization coefficients, and $(k, \rho, \mu_{i,t}, \phi_1, \phi_2)$ are distinct from the model objects

$(k, \rho, \mu(s), \phi)$. Define

$$PV_t(x_i) \equiv \sum_{\tau=1}^{\infty} \rho^{\tau-1} E_t x_{i,t+\tau}. \tag{IA.79}$$

Cho et al.'s (2026) identity can be written (using their notation) as

$$m_{i,t} \approx k + PV_t(\Delta y_i) + \phi_1 PV_t(\mu_i) - \phi_2 PV_t(fci_i) - PV_t(r_i), \tag{IA.80}$$

where $\Delta y_{i,t}$ is output growth, $\mu_{i,t}$ is the log markup, $fci_{i,t}$ is fixed costs and investment scaled by output, and $r_{i,t}$ is the stock-market return. Cho et al. (2026) use a firm-level VAR to construct the expected present values in Eq. (IA.80). Taking the covariance of both sides of the identity with $m_{i,t}$ and dividing by $\mathrm{var}(m_{i,t})$ gives a decomposition:

$$1 \approx \frac{\mathrm{cov}(PV_t(\Delta y_i), m_{i,t})}{\mathrm{var}(m_{i,t})} + \frac{\mathrm{cov}(\phi_1 PV_t(\mu_i), m_{i,t})}{\mathrm{var}(m_{i,t})} - \frac{\mathrm{cov}(\phi_2 PV_t(fci_i), m_{i,t})}{\mathrm{var}(m_{i,t})} - \frac{\mathrm{cov}(PV_t(r_i), m_{i,t})}{\mathrm{var}(m_{i,t})}. \tag{IA.81}$$

Equivalently, each term is the coefficient from a univariate regression of the corresponding (signed) present-value component on $m_{i,t}$. In the within-firm specification, all variables are residualized with respect to firm fixed effects before the covariances are computed. Thus, the decomposition addresses the following question: when a given firm's market value-to-output ratio rises relative to its own average, to what extent is that increase associated with higher expected future markups, higher expected output growth, lower expected fixed costs and investment, or lower expected future returns? The entries are covariance shares and do not necessarily lie between zero and one.

We apply the same logic to model-simulated data from the transition in which we vary $(\phi, \gamma, B, \sigma_A)$. The model counterpart of $M_{i,t}$ is the financial-market value of the firm, based on pricing cash flows at the financial cost of capital; $r_{i,t}$ is the cost of capital. The markup $\mu_{i,t}$ is the model log markup, and the counterpart of Cho et al.'s (2026) fixed-cost-and-investment term is the R&D wage bill divided by sales. We set $\rho = 0.98$ in Eq. (IA.79) as in their analysis, estimate a Cho et al. (2026)-style VAR for $(r, \Delta y, \mu, fci, m)$, construct the implied present-value components, and run the firm-demeaned covariance decomposition corresponding to their Table 2, Panel D. Their empirical VAR also includes leverage, net capital expenditures, asset growth, and market share; we omit these variables, which lack direct counterparts in the simulated panel. Table IA.7 shows that, in the model as in the data, a substantial share of the within-firm variation in firm values is associated with higher expected markups and a lower financial cost of capital.

Table IA.7: **Present-value decomposition of within-firm valuation movements**

	Share of within-firm variation in $m_{i,t}$	
Component	Model transition	Cho et al. (2026) data
Expected cost of capital	37.2	30.7
Expected markups	62.7	36.9
Expected output growth	6.3	51.1
Expected R&D	-10.4	-33.6
Residual	4.2	14.9

This table compares the within-firm present-value decomposition of market value-to-output in Cho et al. (2026) with the analogous decomposition in model-simulated data. The model column applies their decomposition to an annual firm-level panel simulated along the transition generated by the calibrated change in $(B, \phi, \gamma, \sigma_A)$. The data column reports their Table 2, Panel D, which includes firm fixed effects and therefore isolates within-firm variation. Entries are signed covariance shares, $100 \times \text{cov}(\widetilde{PV}^{j}_{i,t}, \widetilde{m}_{i,t})/\text{var}(\widetilde{m}_{i,t})$, where tildes denote residualization with respect to firm fixed effects, $PV^{j}_{i,t}$ is the present-value component in Eq. (IA.80), and $j \in \{\Delta y_i, \mu_i, fci_i, r_i\}$. The residual is one minus the sum of the four reported components and captures approximation error from the log-linear identity and the VAR. All entries are reported in percent.

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